FREE VIBRATION ANALYSIS OF SANDWICH PANEL

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology

in

Mechanical Engineering

By

AMIT KUMAR JHA

Department of Mechanical Engineering
National Institute of Technology
Rourkela

2007

FREE VIBRATION ANALYSIS OF SANDWICH PANEL

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology

in

Mechanical Engineering

By

AMIT KUMAR JHA

Under the guidance of

Prof. N.KAVI

Department of Mechanical Engineering
National Institute of Technology
Rourkela

2007

National Institute of Technology
Rourkela

CERTIFICATE

This is to certify that the thesis entitled "**FREE VIBRATION ANALYSIS OF SANDWICH PANEL**" submitted by Sri Amit Kumar Jha in partial fulfillment of the requirements for the award of Master of Technology Degree in Mechanical Engineering at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Mr. A. Ramamoorthy
Hindustan Aeronautics Limited
Bangalore-560017

Prof. N. Kavi
National Institute of Technology
Rourkela-769008

ACKNOWLEDGEMENT

I would like to express my deep sense of profound gratitude to my guide, Prof. N. Kavi for his guidance and constant support.

I extend my thank to our HOD, Dr. B. K. Nanda for his valuable advices and encouragement.

I would like to thank Mr. A. Ramamoorthy (CM Production, Aircraft Division and Bangalore) for his guidance and constant support.

I would like to thank all HAL officers/employees who are supported me in carrying out the project work in the organization.

I would like to thank to all those who are directly or indirectly supported me in carrying out this thesis work successfully.

Amit Kumar Jha
Roll No. 20503002
Department of Mechanical Engg
National Institute of Technology
Rourkela

CONTENTS

ABSTRACT

Use of Sandwich construction for an aircraft structural component is very common to the present day. One of the primary requirements of aerospace structural materials is that they should have low density, very stiff and strong.

Sandwich panels are thin-walled structures fabricated from two flat sheets separated by a low density core. The core investigated here is of aluminium honeycomb structure because of excellent crush strength and fatigue resistance. Sandwich panels have a very high stiffness to weight ratio with respect equivalent solid plate because of low density core. Modeling is developed in FEA (ANSYS) by consideration of rotary inertia. The free vibration analysis of isotropic plate and sandwich panels are studied. The results of FEA (ANSYS) are compared with results of experimental and analytical work. Eight nodded isoparametric shell element is used for FEA (ANSYS). Convergence study is also included for high accuracy of the results. Analytical results are based on classical bending theory. Mode shapes and corresponding natural frequencies are studied for simply supported sandwich panel. Parameter studies of isotropic plate and sandwich panel are also covered in this analysis. A detailed parameter study has been carried out of a simply supported sandwich panel by increasing the core depth as a percentage of its total thickness, while maintaining a constant mass.

LIST OF FIGURES

LIST OF TABLES

Chapter 1

INTRODUCTION

INTRODUCTION

Sandwich panels have been successfully used for many years in the aviation and aerospace industries, as well as in marine, and mechanical and civil engineering applications. This is due to the attendant high stiffness and high strength to weight ratios of sandwich systems [1]. The use of the sandwich constructions in the aerospace structures can be traced back to Second World War when British De Havilland Mosquito bomber had utilized the sandwich constructions [5]. In the early use, the sandwich structure was very simple in construction, with simple cloth, fabric or thin metal facings were used and soft wood were used as the core.

The conventional sandwich construction comprises a relatively thick core of low-density material which separates top and bottom faceplates (or faces or facings) which are relatively thin but stiff. The materials that have been used in sandwich construction have been many and varied but in quite recent times interest in sandwich construction has increased with the introduction of new materials for use in the facings (e.g. fiber- reinforced composite laminated material) and in the core (e.g. solid foams) [2].

1.1 TYPES OF SANDWICH PANEL

Detailed treatment of the behavior of honeycombed and other types of sandwich panels can be found in monographs by Plantema [3] and Allen [4].

These structures are characterized by a common feature of two flat facing sheets, but the core takes many generic forms; continuous corrugated sheet or a number of discrete but aligned longitudinal top-hat, zed or channel sections (see Figures 1(a)-(e)). The core and facing plates are joined by spot-welds, rivets or self-tapping screws [1].

1.2 SANDWICH CONSTRUCTION

Sandwich construction is a special kind of laminate consisting of a thick core of weak, lightweight material sandwiched between two thin layers (called "face sheets") of strong material figure (1.2). This is done to improve structural strength without a corresponding increase in weight. The choice of face sheet and core materials depends heavily on the performance of the materials in the intended operational environment.

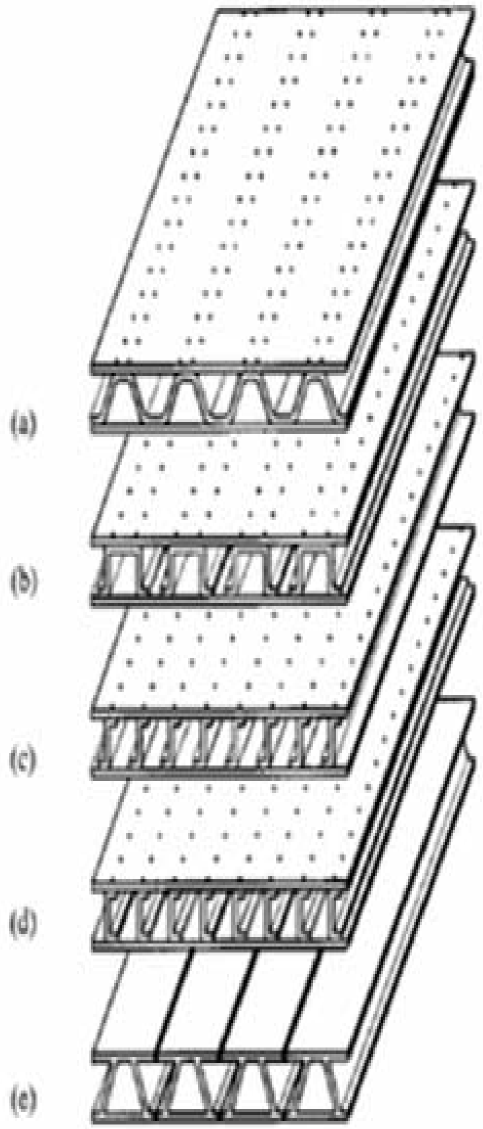


Fig 1.1 Sandwich panel with (a) continuous corrugated-core (b) top-hat core (c) Zed-core (d) channel-core and (e) truss-core

Because of the separation of the core, face sheets can develop very high bending stresses. The core stabilizes the face sheets and develops the required shear strength. Like the web of a beam, the core carries shear stresses. Unlike the web, however, the core maintains continuous support for the face sheets. The core must be rigid enough perpendicularly to the face sheets to prevent crushing and its shear rigidity must be sufficient to prevent appreciable shearing deformations. Although a sandwich composite never has a shearing rigidity as great as that of a solid piece of face-sheet material, very stiff and light structures can be made from properly designed sandwich composites.

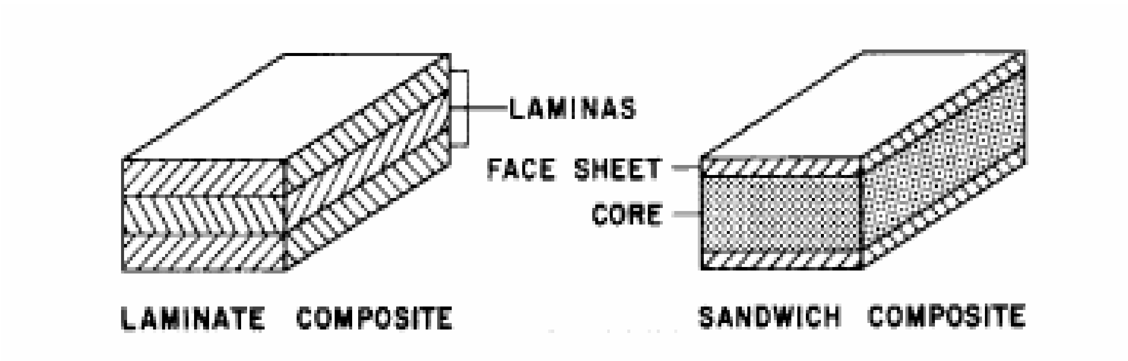


Fig 1.2 Laminate composite and sandwich composite

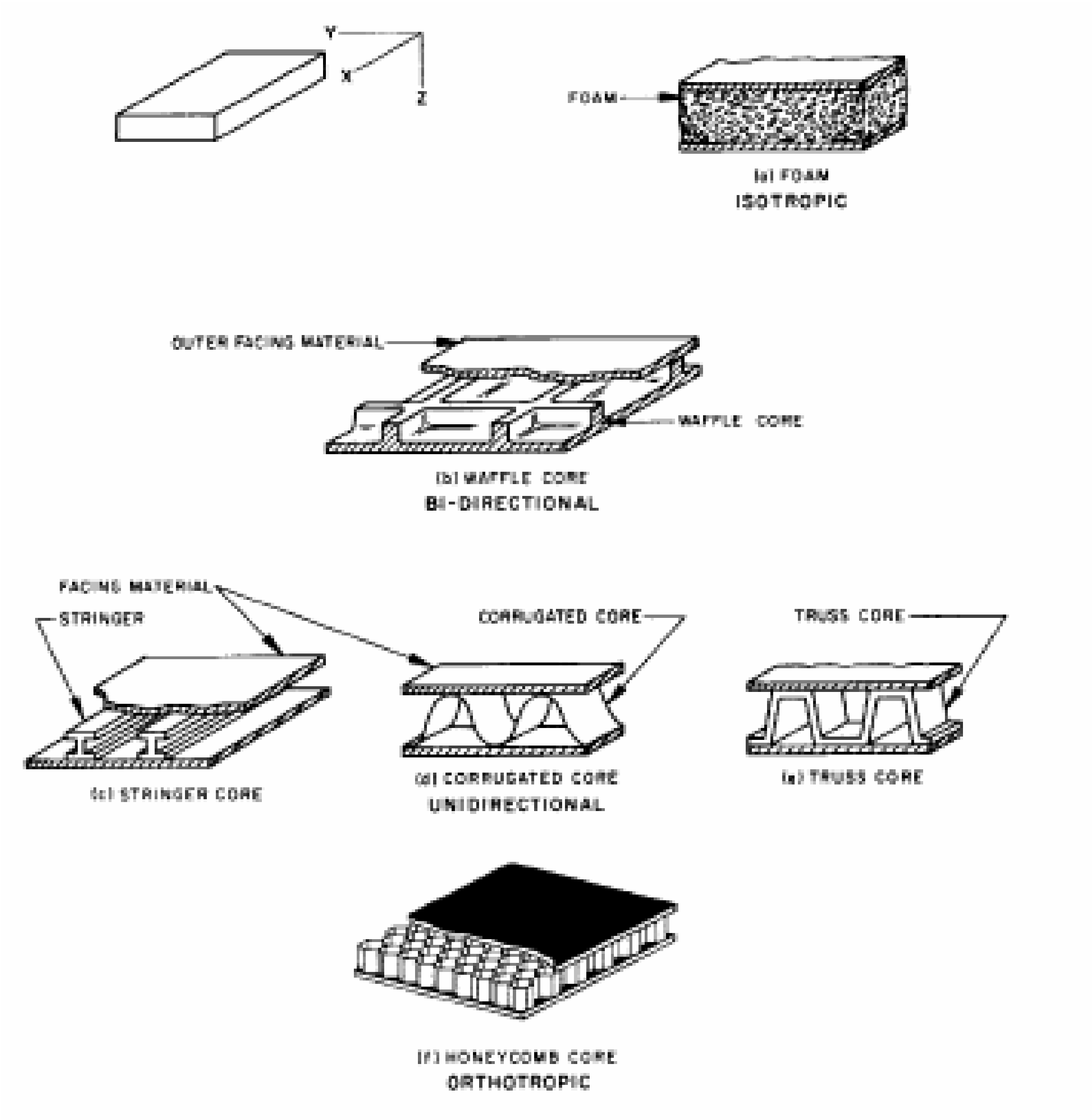


Fig 1.3 Typical sandwich constructions

A useful classification of sandwich composites according to their core properties by respective direction is shown in fig.1.3. To see the core effect upon sandwich strength, let us consider the honeycomb-core and the truss-core sandwich composite.

The honeycomb sandwich has a ratio of shear rigidities in the *xz* and *yz* planes of approximately 2.5 to 1. The face sheets carry in-plane compressive and tensile loads, whereas the core stabilizes the sheets and builds up the sandwich section.

The truss-core sandwich has a shear rigidity ratio of approximately 20 to 1. It can carry axial loads in the direction of the core orientation as well as perform its primary function of stabilizing the face sheets and building up the sandwich section [5].

1.3 PROPERTIES OF MATERIALS USED IN SANDWICH COSNTRUCTION

No single known material or construction can meet all the performance requirements of modern structures. Selection of the optimum structural type and material requires systematic evaluation of several possibilities. The primary objective often is to select the most efficient material and configuration for minimum-weight design [5].

Face Materials

Almost any structural material which is available in the form of thin sheet may be used to form the faces of a sandwich panel. Panels for high-efficiency aircraft structures utilize steel, aluminium or other metals, although reinforced plastics are sometimes adopted in special circumstances. In any efficient sandwich the faces act principally in direct tension and compression. It is therefore appropriate to determine the modulus of elasticity, ultimate strength and yield or proof stress of the face material in a simple tension test. When the material is thick and it is to be used with a weak core it may be desirable to determine its flexural rigidity [4].

Core Materials

A core material is required to perform two essential tasks; it must keep the faces the correct distance apart and it must not allow one face to slide over the other. It must be of low density. Most of the cores have densities in the range 7 to 9.5 lb/ft^3.

Balsa wood is one of the original core materials. It is usually used with the grain perpendicular to the faces of the sandwich. The density is rather variable but the transverse strength and stiffness are good and the shear stiffness moderate.

Modern expanded plastics are approximately isotropic and their strengths and stiffnesses are very roughly proportional to density.

In case of aluminium honeycomb core, all the properties increase progressively with increases in thickness of the foil from which the honeycomb is made [4].

1.4 CURRENT APPLICATIONS

Aerospace Field

In Aerospace industry various structural designs are accomplished to fulfill the required mission of the aircraft. Since a continually growing list of sandwich applications in aircraft/helicopter (example-Jaguar, Light Combat Aircraft, Advanced Light Helicopter) includes fuselages, wings, ailerons, floor panels and storage and pressure tanks as shown in

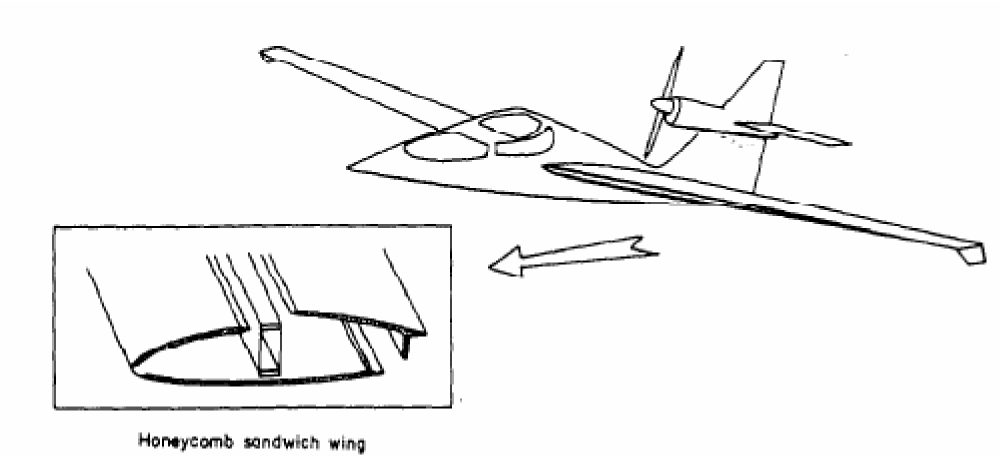
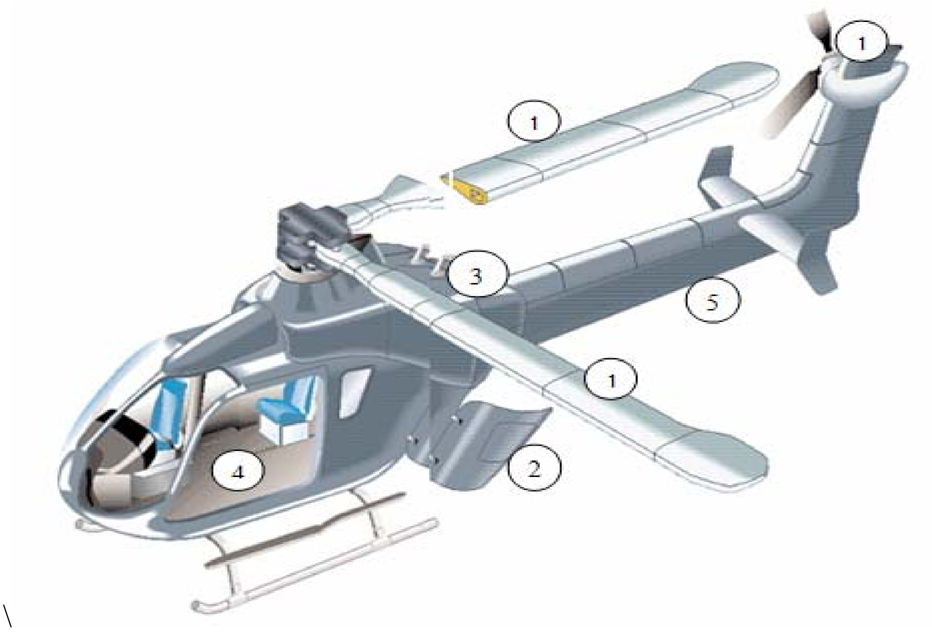


Fig 1.4(a) Application of sandwich structure in aircraft [18].



\

Fig 1.4(b) Application of sandwich structure in helicopter [17].

(1. Rotor Blades, 2. Main and Cargo Doors, 3. Fuselage Panels, 4. Fuselage, 5. Boom and Tail section)

fig (1.4). Honeycomb sandwich structures have been widely used for load-bearing purposes in the aerospace due to their lightweight, high specific bending stiffness and strength under distributed loads in addition to their good energy-absorbing capacity [8]. In a new space-formed system called "Sunflower," the reflector is of honeycomb construction, having a thin coating of pure aluminum protected by a thin coating of silicon oxide to give the very high reflectivity needed for solar-energy collection. Thirty panels fold together into a nose-cone package in the launch vehicle.

Building Construction

Architects use sandwich construction made of a variety of materials for walls, ceilings, floor panels, and roofing. Cores for building materials include urethane foam (slab or foam-in-place), polystyrene foam (board or mold), phenolic foam, phenolic-impregnated paper honeycomb, woven fabrics (glass, nylon, silk, metal, etc.), balsa wood, plywood, metal honeycomb, aluminum and ethylene copolymer foam. Facing sheets can be made from rigid vinyl sheeting (fiat or corrugated) ; glass-reinforced, acrylic-modified polyester**;** acrylic sheeting; plywood; hardwood; sheet metal (aluminum or steel); glass reinforced epoxy; decorative laminate; gypsum; asbestos; and poured concrete [5].

Damped Structures

An increasing number of vibration problems must be controlled by damping resonant response. By using a symmetric sandwich panel with a viscoelastic core, various degrees of damping can be achieved, depending on the core material properties, core thickness, and wavelength of the vibration mode [5].

Chapter 2

LITERATURE SURVEY

LITERATURE SURVEY

2.1 TRUSS CORE SANDWICH

Sandwich panels are thin-walled structures fabricated from two flat sheets, separated by and attached to a core. An analytical solution for the dynamic response of such structures is not available but equivalency in the form of a homogenous orthotropic thick plate can be formulated. This paper considers a truss-core sandwich panel that is similar to conventional sandwich systems, but eliminates many of the attendant problems associated with fabrication of conventional forms. The dynamic response of the truss-core sandwich panel is then formulated as a homogeneous orthotropic thick plate. Closed-form solution for a clamped plate is derived. Closed-form solutions are compared with both 2-D and 3-D finite element results. According to **T.S. LOK** and **Q.H. CHENG**, corrugated form has many advantages over other form of sandwich structure. Some advantages are

- The elimination of discrete connections and all its attendant problems. This is crucial for structures or vessels designed to maintain pressure and water-tightness between the outer and inner environments.
- Better material utilization and ease of manufacturing since only one extrusion process is needed. Transportation, handling and construction cost would be reduced since no large flexible thin sheets are involved.
- Promotion of designer flair to create curved shapes that linear flat conventional sandwich panels are unable to provide [1].

2.2 OPTIMIZATION OF SANDWICH MATERIAL

This article presents an approach to facilitate comparison and optimization of sandwich material combinations. Equivalent homogenized sandwich material properties (bending stiffness, density, and cost) are presented graphically in material selection charts to enable an efficient performance per cost evaluation. The effects of core shear deformations and panel production costs can be included in those sandwich material selection charts. In addition to weight advantages, economical advantages are vital for the potential use of sandwich construction in many applications.

According to **JOCHEN PFLUG AND IGNAAS VERPOEST**, Sandwich construction with a low cost core material can not only be lightweight but also cost effective, especially since the advancement and automation of production processes enable a reduction in production cost for lightweight sandwich panels. However, for low cost applications, sandwich construction is frequently not considered because of limited knowledge about their cost-saving potential. It is thus important to provide tools to evaluate and present the cost-saving potential of sandwich material combinations already in the stage of materials selection. For a given sandwich material combination, an optimization, including shear deformations of the core as well as optimizations toward a maximum bending strength, involving different failure modes, can be performed [6].

2.3 GLOBAL HIGHER THEORY

Laminated composite and sandwich plates are being increasingly used in advanced aerospace structures because they can exhibit many favorable characteristics such as high specific modulus and strength and low specific density. To use them efficiently, it is necessary to develop appropriate models capable of accurately predicting their structural and dynamical behavior.

Due to ignoring the transverse shear deformation and overestimating the natural frequency, the classical laminate plate theory becomes inadequate for the analysis of thick laminated and sandwich plates. Therefore it is necessary to consider the effect of transverse shear deformation in the study of thick laminated structures. To take into account the effects of shear deformation, the first-order shear deformation theories are firstly developed whereas the accuracy of solutions of this theory will be strongly dependent on the shear correction factors. In order to overcome the limitations of first-order shear deformation theory, the global higher-order theories that include higher-order terms in Taylor’s expansions of the displacement in the thickness direction. However, by further research, it is found that the global higher-order theories also overestimate natural frequency for laminated composite plates with different thickness and materials at each ply and soft-core sandwich plates because these higher-order theories violate continuity conditions of the transverse stress components.

To overcome the limitations of the global higher-order theories and layerwise theories, this paper is to use the global–local theory to predict dynamical response of laminated composite plates with arbitrary layout and soft-core sandwich plates. The global–local higher-order theory is firstly developed by Li and Liu and further study on the global–local theory has been presented by **Wu and Chen.** This theory possesses the accuracy of layer wise theory and efficiency of global higher-order theory, moreover, which satisfies displacements and transverse shear stresses continuity conditions at the interfaces.

Natural frequencies of laminated composite and sandwich plates have been calculated by using the global–local higher-order theory, and these results are compared with those previously published. These comparisons revealed that the present theory can accurately predict natural frequencies of general laminated plates. Moreover, this theory is still suitable for dynamical problems of laminated composite plates with variational thickness and materials at each layer and soft-core sandwich plates. However, numerical results show that for these special structures, the global higher- and first-order theories that violate continuity of interlaminar stresses will encounter some difficulties and overestimate the natural frequencies [7].

Chapter 3

FINITE ELEMENT FORMULATION

FINITE ELEMENT FORMULATION

3.1 THEORY

The solution of a real life problem involving an arbitrary plate geometry and complicated loading and boundary conditions cannot be easily realized using analytical methods. A numerical analysis technique, especially finite element analysis method, is suited most to solve such problems.

The finite element method is essentially a piecewise application of a variational method. The finite element formulation is based on the conventional Rayleigh-Ritz method and Galerkin Method. In the Rayleigh-Ritz method we construct a functional that expresses the total potential energy π of the system in terms of nodal variables d_i. The problem is solved using the stationary-functional conditions $\partial\pi/\partial d_i = 0$ [9].

In the case of the Galerkin method, the functional for the residual R is formed using differential equations of the residual R to zero. In structural mechanics, both the Rayleigh-Ritz method and the Galerkin method yield identical results, when both use the same field variables.

The Classical Plate Theory evaluates the behavior of the thin isotropic plate, assuming that the normal to the undeformed middle plane remains straight, normal and inextensible during the deformation [14]. But same assumption is not valid for a laminated sandwich plate as the transverse shear deformation and elastic coupling effects become more predominant and deformation of the core plays a vital role in overall deformation of the plate. Analysis of a sandwich plate calls for inclusion of the shear deformation by splitting the total deflection into deflection due to bending and due to shear deformation of the core. The improvement in the classical plate theory due to Mindlin and Reissner, takes into account the effects of shear deformation, in addition an arbitrary shear correction factor to the transverse shear stiffness is also applied in order to account for the non uniform shear distribution at any cross section.

3.2 SHEAR DEFORMATION OF THE PLATE

The theory proposed by Mindlin assumes that straight lines originally normal to the mid surface, before the deformation , remains straight but not normal to the deformed mid surface i.e. the average rotation of the section may be taken as the rotation in which normal remains perpendicular to the mid surface plus an additional rotation due to transverse shear. Thus the actual shear deformation is assumed to be equivalent to a straight line representing a uniform shear strain through the thickness [12]. This theory makes following assumptions:-

(a) The lateral deflections ‘w’ are small.

(b) Normal to the plate mid surface before deformation remains straight but not necessary normal to it after deformation.

(c) Stresses normal to the mid surface are negligible

In this theory, the second assumption differs from the Classical Plate Theory, while other two are same for both the theories. Fig 3.1(a) in which Φ_x, denotes an average transverse shear strain for a section y = constant, the total rotation θ_x, can be express as follows:-

$$\theta_x = \frac{dw}{dx} + \Phi_x \quad \textbf{...(3.1)}$$

where dw/dx is the rotation of the element along the center line due to bending only.
Similarly for the section x = constant (fig 2.1(b))

$$\theta_y = \frac{dw}{dy} + \Phi_y \quad \textbf{... (3.2)}$$

Hence the average shear strain Φ_x and Φ_y are given by following relations:-

$$\Phi_x = \theta_x - \frac{dw}{dx}$$
$$\Phi_y = \theta_y - \frac{dw}{dy} \quad \textbf{... (3.3)}$$

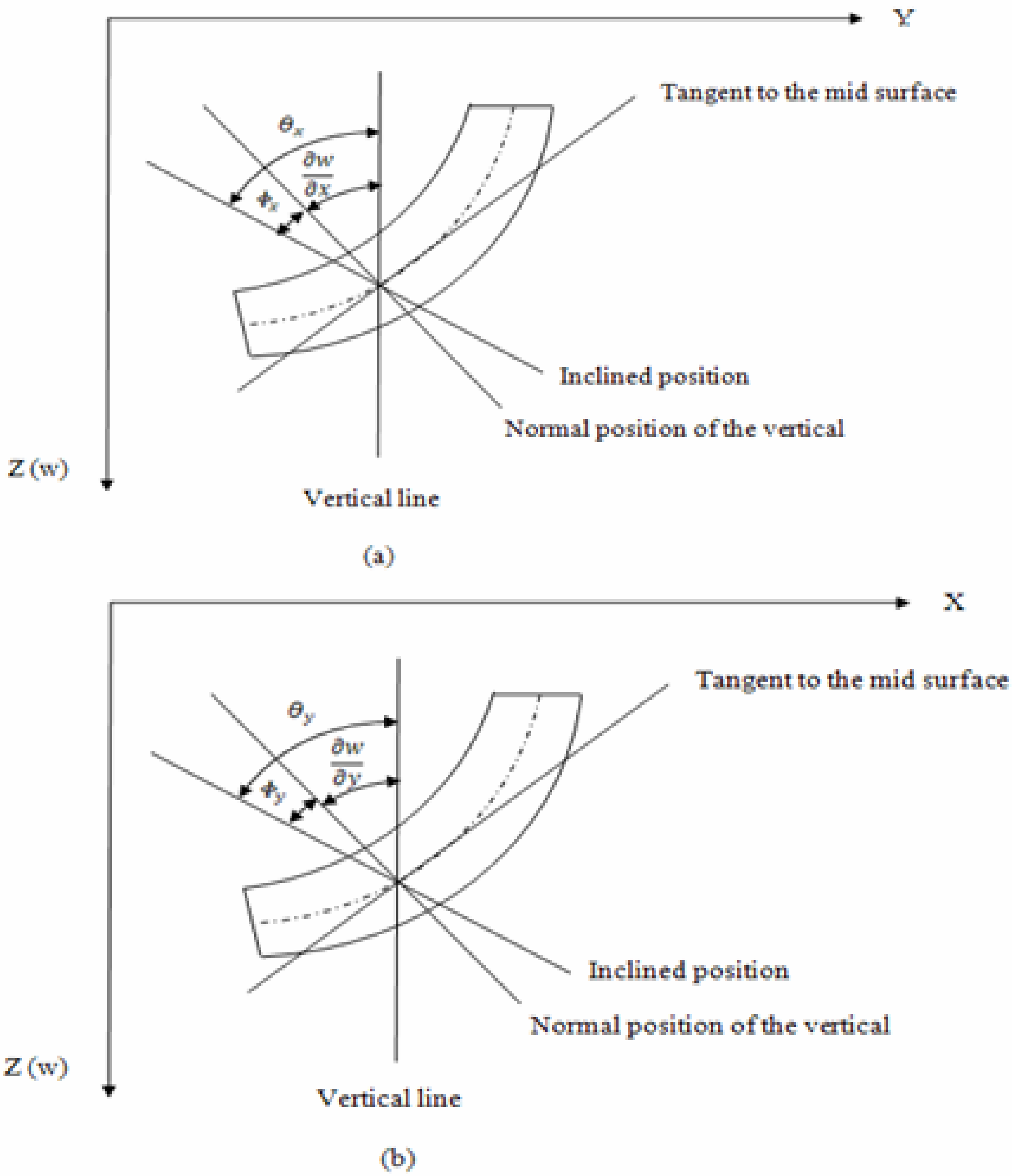


Fig 3.1 Mindlin's plate theory

The kinematics relation can be expressed as

$$\{k\} = \{k_{xx} \quad k_{yy} \quad k_{xy}\}^T$$
$$= \{-\theta_{x,x} \quad -\theta_{y,y} \quad -(\theta_{x,y} + \theta_{y,x})\}^T$$
$$\{\gamma\} = \left\{-\theta_x + \frac{dw}{dx} \quad -\theta_y + \frac{dw}{dy}\right\}^T \qquad \text{...(3.4)}$$

where { k } are the plate curvatures,{ θ } are the planar relations,{ γ } are the shear strain and ‘w’ are the lateral deflections of the plate.
The bending and shear behavior of an anisotropic plate are given by [11]: -

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} K_{xx} \\ K_{yy} \\ K_{xy} \end{Bmatrix}$$

and

$$\begin{Bmatrix} Q_{xz} \\ Q_{yz} \end{Bmatrix} = Cs \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \qquad \text{...(3.5)}$$

where Cs shear correction factor. It is a function of cross sectional shape and Poisson’s ratio (ν).

The bending and shear strain energies are expressed as follows:-

$$U_b = \frac{1}{2}\int \{K\}^T[D]\{K\}\,dA$$

and $\qquad$ ... (3.6)

$$U_s = \frac{1}{2}\int \{\gamma\}^T[A]\{\gamma\}dA$$

where

$$[D] = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}$$

and

$$[A] = C_s \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix}$$

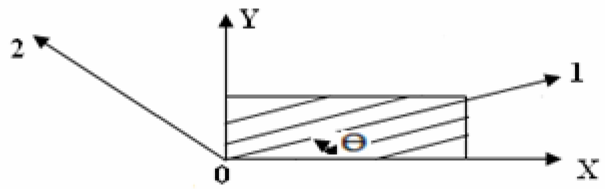


Fig 3.2 (a) Anisotropic Lamina

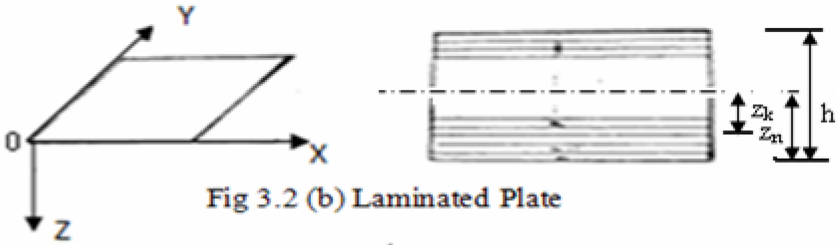


Fig 3.2 (b) Laminated Plate

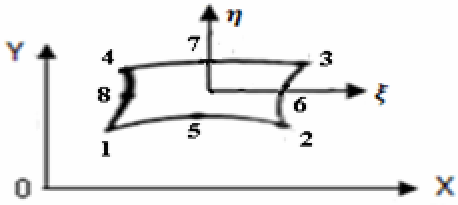


Fig 3.3 Eight Nodded Isoparametric shell Element

3.3 CONSTITUTIVE EQUATION FOR ANISOTROPIC PLATES

Consider a unidirectional lamina as depicted in the fig. 3.2. Neglecting the normal stresses perpendicular to the plane of the lamina, the stress-strain relations in the principal material direction 1, 2 and 3 are as follows [11]:-

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix}$$

and

$$\begin{Bmatrix} \tau_{13} \\ \tau_{23} \end{Bmatrix} = Cs \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{13} \\ \gamma_{23} \end{Bmatrix} \qquad \text{...(3.7)}$$

where

$$Q_{11} = {}^{E_{11}}\!/\!(1 - \nu_{12}\nu_{21})$$

$$Q_{12} = {}^{\nu_{12}E_{22}}\!/\!(1 - \nu_{12}\nu_{21})$$

$$Q_{22} = {}^{E_{22}}\!/\!(1 - \nu_{12}\nu_{21})$$

$$Q_{44} = G_{13},\ Q_{55} = G_{23},\ Q_{66} = G_{12}$$

The stress-strain relations of the lamina with respect to the x, y and z axes are as follows:-

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \quad \textbf{... (3.8)}$$

$$\begin{Bmatrix} \overline{Q_{11}} \\ \overline{Q_{22}} \\ \overline{Q_{12}} \\ \overline{Q_{66}} \\ \overline{Q_{16}} \\ \overline{Q_{26}} \end{Bmatrix} = \begin{bmatrix} m^4 & n^4 & 2m^2n^2 & 4m^2n^2 & -4m^3n & -4m^3n \\ n^4 & m^4 & 2m^2n^2 & 4m^2n^2 & 4mn^3 & 4m^3n \\ m^2n^2 & m^2n^2 & m^4+n^4 & -4m^2n^2 & 2(m^3n-mn^3) & 2(mn^3-m^3n) \\ m^2n^2 & m^2n^2 & -2m^2n^2 & (m^2-n^2)^2 & 2(m^3n-mn^3) & 2(mn^3-m^3n) \\ m^3n & -mn^3 & mn^3-m^3n & 2(mn^3-m^3n) & m^4-3m^2n^2 & 3m^2n^2-n^4 \\ mn^3 & -m^3n & m^3n-mn^3 & 2(m^3n-mn^3) & 3m^2n^2-n^4 & m^4-3m^2n^2 \end{bmatrix} \begin{Bmatrix} Q_{11} \\ Q_{22} \\ Q_{12} \\ Q_{66} \\ Q_{16} \\ Q_{26} \end{Bmatrix} \quad \textbf{... (3.9)}$$

$$\mathrm{m} = \cos\theta \quad \text{and} \quad n = \sin\theta \qquad \textbf{... (3.10)}$$

3.4 CONSTITUTIVE RELATIONS FOR THE LAMINATED PLATE

Consider a laminated plate having width w and thickness h. The plate is made of n number of laminae. The interface between the lamina is very thin and they do not deform under shear. Thus the displacements are continuous through the thickness of the laminate. The linear plain strains at a distance z from the mid surfaces are given as follows [10]:-

$$\varepsilon_{xx} = u_{,x} = \varepsilon^0_{xx} + zk_{xx}$$

$$\varepsilon_{yy} = v_{,y} = \varepsilon^0_{yy} + zk_{yy}$$

$$\gamma_{xy} = u_{,x} + v_{,y} = \gamma^0_{xy} + zk_{xy} \qquad \text{... (3.11)}$$

where ε^0_{xx}, ε^0_{yy} and γ^0_{xy} are the mid plane strains. These are defined by following expressions:-

$$\varepsilon^{0}_{xx} = u^{0}_{,x}$$

$$\varepsilon^{0}_{yy} = v^{0}_{,x}$$

and $$\gamma^{0}_{xy} = u^{0}_{,y} + v^{0}_{,x}$$

where u^0, v^0 are the mid plane deformations in the x and y directions respectively and k_{xx}, k_{yy} and k_{xy} are the curvatures . Therefore

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon^{0}_{xx} \\ \varepsilon^{0}_{yy} \\ \gamma^{0}_{xy} \end{Bmatrix} + z \begin{Bmatrix} k_{xx} \\ k_{yy} \\ k_{xy} \end{Bmatrix} \qquad \text{... (3.12)}$$

since the transverse shear deformation is assumed uniform across the thickness of the laminate γ_{xz} and γ_{yz} are identical to φ_x and φ_y respectively i.e.

$$\gamma_{xz} = \varphi_x \quad \text{and} \quad \gamma_{yz} = \varphi_y \qquad \text{... (3.13)}$$

thus

$$\begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = Cs \begin{bmatrix} \overline{Q_{44}} & \overline{Q_{45}} \\ \overline{Q_{45}} & \overline{Q_{55}} \end{bmatrix} \begin{Bmatrix} \varphi_x \\ \varphi_y \end{Bmatrix} \qquad \text{... (3.14)}$$

The internal force and moment resultants of the laminate are obtained by integrating elemental forces and moments over the thickness of the laminate**.**

$$\{N_{xx} \quad N_{yy} \quad N_{xy}\}^T = \int_{-h/2}^{+h/2} \{\sigma_{xx} \quad \sigma_{yy} \quad \tau_{xy}\}^T_K \, dz \qquad \text{... (3.15)}$$

$$\{M_{xx} \quad M_{yy} \quad M_{xy}\}^T = \int_{-h/2}^{+h/2} \{\sigma_{xx} \quad \sigma_{yy} \quad \tau_{xy}\}^T_K \, dz \qquad \text{... (3.16)}$$

$$\{Q_{xz} \quad Q_{yz}\}^T = \int_{-h/2}^{+h/2} \{\tau_{xz} \quad \tau_{yz}\}^T_K \, dz \qquad \text{... (3.17)}$$

Equations (3.15) through (3.17) can be represented as follows:-

$$\{P\} = [D]\{\varepsilon\} \qquad \dots(3.18)$$

where

$$\{P\} = \{N_{xx} \quad N_{yy} \quad N_{xy} \quad M_{xx} \quad M_{yy} \quad M_{xy} \quad Q_{xz} \quad Q_{yz}\}^T$$

$$\{\varepsilon\} = \{\varepsilon_{xx} \quad \varepsilon_{yy} \quad \gamma_{xy} \quad K_{xx} \quad K_{yy} \quad K_{xy} \quad \Phi_{xz} \quad \Phi_{yz}\}^T$$

$$[D] = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{55} \end{bmatrix}$$

and

$$(A_{ij} \quad B_{ij} \quad D_{ij}) = \sum_{k=1}^{n} \int_{Z_{k-1}}^{Z_k} [\overline{Q_{ij}}]_k (1 \quad Z \quad Z^2)\, dz \qquad \dots(3.19)$$

where $i, j = 1, 2, 6$

3.5 FINITE ELEMENT FORMULATION

3.5.1 EIGHT NODDED ISOPARAMETRIC ELEMENT

In the analysis of structural problems of complex shapes involving curved boundaries, employing simple triangular or rectangular element does not provide accurate results. The concept of isoparametric element is based on the transformation of the parent element from the local or natural coordinate system to an arbitrary shape in Cartesian coordinate system. A convenient way of expressing the transformation is to make use of the shape functions of the rectilinear elements in their natural coordinate system and the nodal values of the coordinates [10].

In the analysis of the plate element, at any point, there are five independent displacement quantities u, v, w, θx and θy being considered for inclusion of shear deformation. Figure 3.3 shows an eight nodded isoparametric shell element. The geometry of the element is described by the following relation:-

$$x = \sum_{i=0}^{8} N_i x_i \quad \text{and} \quad y = \sum_{i=0}^{8} N_i y_i \qquad \dots(3.20)$$

The variations of the displacements in an element are as follows:-

$$u = \sum_{i=0}^{8} N_i u_i \qquad v = \sum_{i=0}^{8} N_i v_i \qquad w = \sum_{i=0}^{8} N_i w_i$$

$$\theta x = \sum_{i=0}^{8} N_i \theta_{xi} \qquad \theta y = \sum_{i=0}^{8} N_i \theta_{yi} \qquad \textbf{... (3.21)}$$

where N_i are the shape functions, x_i and y_i are the nodal coordinates, ($u_{i,}$ v_i and w_i) are the nodal displacements along x, y, z directions, θx and θy are the nodal slopes along x, y directions.

The shape functions are as follows:-

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)(-\xi-\eta-1)$$

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1)$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)(\xi-\eta-1)$$

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)(-\xi+\eta-1)$$

$$N_5 = \frac{1}{2}(1+\xi)(1-\xi)(1-\eta)$$

$$N_6 = \frac{1}{2}(1+\xi)(1+\eta)(1-\eta)$$

$$N_7 = \frac{1}{2}(1+\xi)(1+\eta)(1-\xi) \qquad \textbf{... (3.22)}$$

$$N_8 = \frac{1}{2}(1-\xi)(1+\eta)(1-\eta)$$

3.5.2 DISPLACEMENT ANALYSIS

The displacements {u} within an element are usually expressed as [9]

$$\{u\} = [N]\ \{d_e\} \qquad \textbf{...(3.23)}$$

where [N] is shape function and {d} is the element nodal displacements with respect to the local axis. The strains {ε} in an element are defined in terms of the displacements as

$$\{\varepsilon\} = [\Delta]\ \{u\} \qquad \textbf{... (3.24)}$$

where [Δ] is differential operator.

Strain-nodal displacement relations are obtained by combining equations (3.23) and (3.24):-

$$\{\varepsilon\} = [B]\{d_e\} \quad \ldots (3.25)$$

where $$[B] = [\Delta][N] \quad \ldots (3.26)$$

Stress-strain relation in an element is expressed as

$$\{\sigma\} = [Q]\{\varepsilon\} \quad \ldots (3.27)$$

where [Q] is the elastic stiffness.

Hence the stress-nodal displacement relations are obtained as

$$\{\sigma\} = [Q]\,[B]\{d_e\} \quad \ldots (3.28)$$

The total potential of an element is first computed to apply the Rayleigh-Ritz variational approach. The total potential of the element is given by

$$\pi_e = \frac{1}{2}\int_{Ve}\{\sigma\}^T\{\varepsilon\}\,dV - \int_{Se}\{u\}^T\{q\}\,dS \quad \ldots (3.29)$$

where $\{q\}$ are the surface tractions.

From above equations, we get

$$\pi_e = \frac{1}{2}\int_{Ve}\{d_e\}^T[B]^T\,[Q]\,[B]\,\{d_e\}\,dV - \int_{Se}\{d_e\}^T[N]^T\,\{q\}\,dS \quad \ldots (3.30)$$

Applying the principle of Minimum Potential Energy yields

$$[K_e]\{d_e\} = \{P_e\} \quad \ldots (3.31)$$

where $[K_e]$ is element stiffness matrix and $\{P_e\}$ are the element nodal forces. These are expressed as

$$[K_e] = \int_{Ve}[B]^T\,[Q]\,[B]\,dV \quad \text{and} \quad \{P_e\} = \int_{Se}[N]^T\,\{q\}\,dS \quad \ldots (3.32)$$

Transformation of eq. (3.31) from the local axes to the global axes and proper assembly of terms over all elements will lead to a set of equilibrium equations for the complete structure, as

$$[K]\{d\} = \{P\} \quad \ldots (3.33)$$

where [K], {d} and {P} correspond to the global axes.

According to Galerkin weighted residual approach, the residual equation is expressed as

$$\int_{Ve} [N]^T \left[(Q_{ijkl} \quad \varepsilon_{kl})_{,\,j} - q_i \right] dV = 0 \qquad \ldots(3.34)$$

where $(Q_{ijkl} \quad \varepsilon_{kl})_{,\,j} - q_i = 0$ are equilibrium equations and [N] are the weight functions. Applying the Green-Gauss's Theorem to eq. (3.34) and expanding-

$$\int_{Ve} [B]^T (Q_{ijkl})[B]\{d_e\}\, dV - \int_{Se} [N]^T \{q_i\} dS = 0 \qquad \ldots(3.35)$$

After eliminating the non-essential boundary conditions, eq. (3.35) can be written in the form as given by eq. (3.33).

3.6 VIBRATION SOLUTION TECHNIQUE

Using isoparametric finite element concept, strain nodal displacement relations for the element (in matrix form) is expressed as

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ K_{xx} \\ K_{yy} \\ K_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \sum_{i=1}^{8} \begin{bmatrix} \frac{dN_i}{dx} & 0 & 0 & 0 & 0 \\ 0 & \frac{dN_i}{dy} & 0 & 0 & 0 \\ \frac{dN_i}{dy} & \frac{dN_i}{dx} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{dN_i}{dx} & 0 \\ 0 & 0 & 0 & 0 & -\frac{dN_i}{dy} \\ 0 & 0 & 0 & -\frac{dN_i}{dy} & -\frac{dN_i}{dx} \\ 0 & 0 & \frac{dN_i}{dx} & -N_i & 0 \\ 0 & 0 & \frac{dN_i}{dy} & 0 & -N_i \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_{xi} \\ \theta_{yi} \end{Bmatrix} \qquad \ldots(3.36)$$

The element stiffness matrix is given by

$$[K_e] = \iint [B]^T \; [D]\,[B]\, dx\, dy \qquad \ldots(3.37)$$

where [B] is the strain displacement matrix given in eq. (3.36) and [D] is the constitutive matrix given in eq. (3.18). The total element stiffness matrix $[K_e]$ is the sum of bending stiffness matrix $[K_b]$ and transverse shear stiffness matrix $[K_s]$.

$$[K_e] = [K_b] + [K_s] \qquad \ldots(3.38)$$

The element mass matrix is expressed as

$$[M_e] = \iint [N]^T \; [P]\,[N]\, dx\, dy \qquad \ldots(3.39)$$

where

$$[N] = \sum_{i=1}^{8} \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix} \quad \dots (3.40)$$

and

$$[P] = \sum_{i=1}^{8} \begin{bmatrix} P & 0 & 0 & 0 & 0 \\ 0 & P & 0 & 0 & 0 \\ 0 & 0 & P & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \quad \dots (3.41)$$

with

$$P = \sum_{k=1}^{n} \int_{Z_{K-1}}^{Z_K} \rho \, dz \quad \text{and} \quad I = \sum_{k=1}^{n} \int_{Z_{K-1}}^{Z_K} z^2 \rho \, dz \quad \dots (3.42)$$

Here ρ is mass density.

The stiffness matrix $[K_e]$ and the mass matrix $[M_e]$ are evaluated first by expressing the integrals in the local isoparametric coordinates ξ and η of the element and then performing numerical integration employing the 2x2 Gauss Quadrature .The element matrices are assembled after performing appropriate transformations. Thus one obtains for the static case

$$[K]\,\{d\} = \{P\} \quad \dots (3.43)$$

and for free vibration case

$$|[K] - \omega_{mn}^2 [M]| = 0 \quad \dots (3.44)$$

where ω_{mn} is frequency of the laminated plate.

Chapter 4

MACROMECHANICAL BEHAVIOUR OF LAMINATES

MACROMECHANICAL BEHAVIOUR OF LAMINATES

4.1 INTRODUCTION

The main constituents of a structural composite are the reinforcements and the reinforcements and the matrix. The reinforcements, which are stronger and stiffer, are dispersed in a comparatively less strong and stiff matrix material. The reinforcements share the major load and in some cases especially when a composite consists of fiber reinforcements dispersed in a weak matrix (e.g.-carbon/epoxy composite), the fibers carry almost all the load. The strength and stiffness properties of are therefore controlled by strength and stiffness of constituent fibers. The matrix also shares the load when there is not much difference between the strength and stiffness properties of reinforcements and matrix (e.g.-SiC/Titanium composite). For example to obtain a good conducting composite with SiC fibers, aluminum matrix is a better option due to high thermal conductivity than titanium matrix [11].

4.2 MACROMECHANICAL BEHAVIOR

4.2.1 INTRODUCTION

The heterogeneity in a composite material is introduced due to not only its bi-phase or multi-phase composition, but also laminations. This leads to a distinctly different stress strain behavior in the case of laminates. The anisotropy caused due to fiber orientations and the resulting extension-shear and bending-twisting coupling as well as the extension-bending coupling developed due to unsymmetric lamination adds to the complexities [13].

4.2.2 UNIDIRECTIONAL LAMINA

The stress-strain relations for a unidirectional lamina with two-dimensional anisotropy (fig 4.1(a)-off-axis lamina), are expressed as

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} \qquad \ldots(\mathbf{4.1})$$

or $$\{\sigma\} = [Q_{ij}]\{\varepsilon\} \qquad i, j = 1, 2, 6 \qquad \ldots(\mathbf{4.2})$$

where $[Q_{ij}]$ are the reduced stiffnesses (elastic constants) for plane stress.

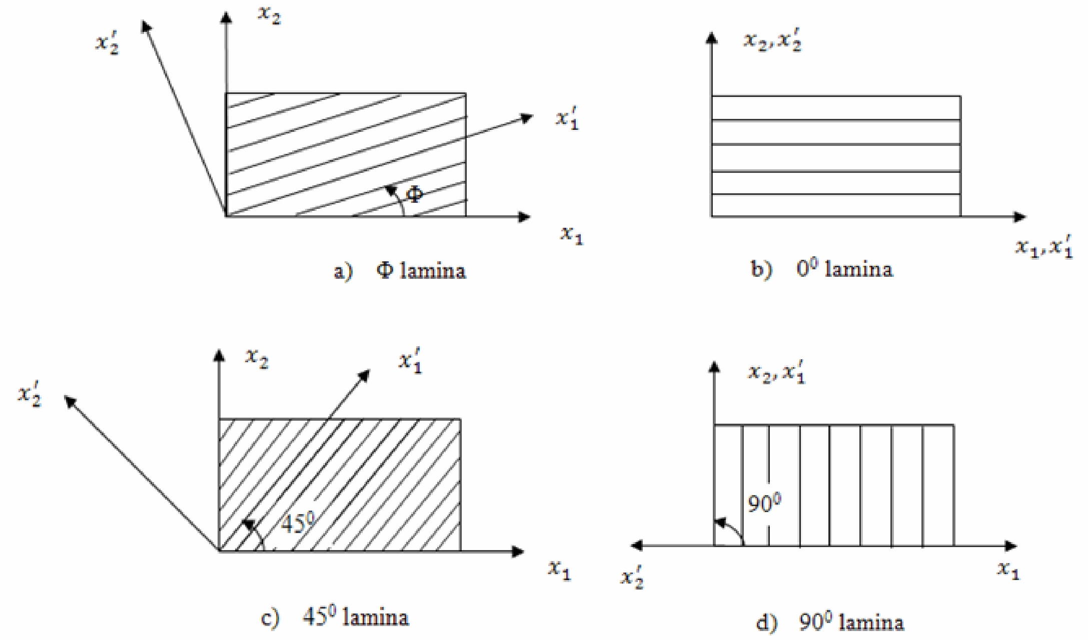


Fig 4.1 Unidirectional Laminae

Similarly, in terms of compliances, the stress strain relations are

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{21} & S_{22} & S_{26} \\ S_{16} & S_{26} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} \qquad \ldots(\mathbf{4.3})$$

or $\{\varepsilon\} = [S_{ij}]\{\sigma\}$ i, j =1, 2, 6 ...(**4.4**)

For the case of an on-axis lamina with two-dimensional orthotropy (fig 4.1), the stress-strain relations are

$$\begin{Bmatrix} \sigma'_1 \\ \sigma'_2 \\ \sigma'_6 \end{Bmatrix} = \begin{bmatrix} Q'_{11} & Q'_{12} & 0 \\ Q'_{21} & Q'_{22} & 0 \\ 0 & 0 & Q'_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon'_1 \\ \varepsilon'_2 \\ \varepsilon'_6 \end{Bmatrix} \qquad \ldots(\mathbf{4.5})$$

$$\begin{Bmatrix} \varepsilon'_1 \\ \varepsilon'_2 \\ \varepsilon'_6 \end{Bmatrix} = \begin{bmatrix} S'_{11} & S'_{12} & 0 \\ S'_{21} & S'_{22} & 0 \\ 0 & 0 & S'_{66} \end{bmatrix} \begin{Bmatrix} \sigma'_1 \\ \sigma'_2 \\ \sigma'_6 \end{Bmatrix} \qquad \ldots(\mathbf{4.6})$$

where

$$Q'_{11} = \frac{E_{11}}{(1 - \nu'_{12}\nu'_{21})} \quad ; \quad Q'_{22} = \frac{E_{22}}{(1 - \nu'_{12}\nu'_{21})} \qquad \text{... (4.7)}$$

$$Q'_{12} = Q'_{21} = \frac{\nu'_{21} E'_{11}}{(1 - \nu'_{12}\nu'_{21})} = \frac{\nu'_{12} E'_{22}}{(1 - \nu'_{12}\nu'_{21})} \quad ; \quad Q'_{66} = G'_{21} \qquad \text{... (4.8)}$$

and

$$S'_{11} = \frac{1}{E'_{11}} \quad ; \quad S'_{22} = \frac{1}{E'_{22}} \qquad \text{... (4.9)}$$

$$S'_{12} = S'_{22} = \frac{-\nu'_{12}}{E'_{11}} = \frac{-\nu'_{21}}{E'_{22}} \quad ; \quad S'_{66} = \frac{1}{G'_{21}} \qquad \text{... (4.10)}$$

In the above equations, the engineering constants ($E'_{11}, E'_{22}, \nu'_{12}, \nu'_{21}$ and G'_{12}) are referred to the orthotropic axis system (X'_1, X'_2) [11].

4.2.3 TRANSFORMATION OF ELASTIC CONSTANTS

If the elastic constants and compliances of a material are known with respect to a given co-ordinate system, then the corresponding values with respect to any other mutually perpendicular coordinate can be determined using transformation rules[13].

Two dimensional cases

If the elements of $[Q_{ij}]$ and $[Q'_{ij}]$ refer to the (X'_1, X'_2) and (x_1, x_2) coordinates respectively, then transformation laws for reduced elastic constants are obtained as follows [11]:

$$[Q_{ij}] = [T_\varepsilon]^T \, [Q'_{ij}] \, [T_\varepsilon] \qquad \text{... (4.11)}$$

$$[Q'_{ij}] = [T_\sigma] \, [Q_{ij}] \, [T_\sigma]^T \qquad \text{... (4.12)}$$

where $[T_\varepsilon]$ and $[T_\sigma]$ are defined as

$$[T_\varepsilon]=\begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2-n^2 \end{bmatrix} \qquad \textbf{... (4.13)}$$

$$[T_\sigma]=\begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{bmatrix} \qquad \textbf{... (4.14)}$$

where m = cosΦ and n = sinΦ

From equation (4.11) and (4.13):

$$\begin{Bmatrix} Q_{11} \\ Q_{22} \\ Q_{12} \\ Q_{66} \\ Q_{16} \\ Q_{26} \end{Bmatrix} = \begin{bmatrix} m^4 & n^4 & 2m^2n^2 & 4m^2n^2 \\ n^4 & m^4 & 2m^2n^2 & 4m^2n^2 \\ m^2n^2 & m^2n^2 & m^4+n^4 & -4m^2n^2 \\ m^2n^2 & m^2n^2 & -2m^2n^2 & (m^2-n^2)^2 \\ m^3n & -mn^3 & mn^3-m^3n & 2(mn^3-m^3n) \\ mn^3 & -m^3n & m^3n-mn^3 & 2(m^3n-mn^3) \end{bmatrix} \begin{Bmatrix} Q'_{11} \\ Q'_{22} \\ Q'_{12} \\ Q'_{66} \end{Bmatrix} \qquad \textbf{... (4.15)}$$

4.2.4 GENERAL LAMINATES

Let us consider a general thin laminate of thickness h (fig. 4.2). The laminate consists of n number of unidirectional and/or bidirectional laminae, where each lamina may be of different materials and thicknesses and have different fibre orientations (Φ). A thin general laminate is essentially a two-dimensional problem, but cannot be treated as a two-dimensional plane stress problem as has been done for a unidirectional lamina. The existence of extension-bending coupling causes bending, even if the laminate is subjected to inplane loads only. Therefore, thin plate bending theories are employed in derivation of constitutive relations. Kirchhoff's assumptions related to the thin plate bending theory

are applicable for constitutive relations [13].

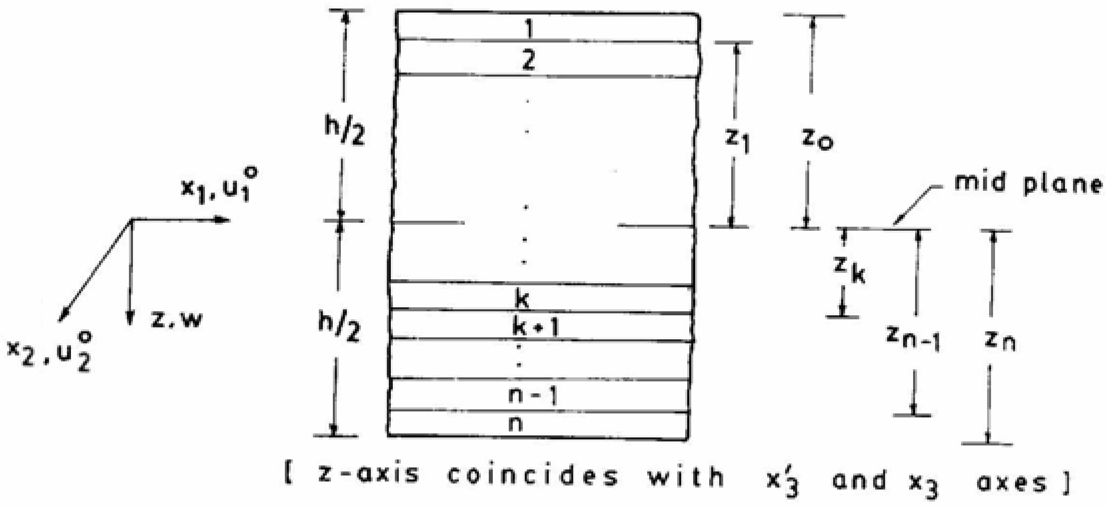


Fig 4.2 General Laminates

Let u_1^0, u_2^0 and w are the mid-plane displacements, and w is constant through the thickness of the lamina. Then the mid-plane strains are given by

$$\varepsilon_1^0 = \frac{\partial u_1^0}{\partial x_1}, \quad \varepsilon_2^0 = \frac{\partial u_2^0}{\partial x_2} \quad \text{and} \quad \varepsilon_6^0 = \frac{\partial u_1^0}{\partial x_2} + \frac{\partial u_2^0}{\partial x_1} \qquad \dots(4.16)$$

and the curvatures, which are constant through the thickness of the laminate, are expressed as

$$k_1 = -\frac{\partial^2 w}{\partial x_1^2}, \quad k_2 = -\frac{\partial^2 w}{\partial x_2^2} \quad \text{and} \quad k_6 = -2\frac{\partial^2 w}{\partial x_1\, \partial x_2} \qquad \dots(4.17)$$

The strains at any distance z are then given as

$$\varepsilon_1(z) = \varepsilon_1^0 + zk_1\,, \quad \varepsilon_2(z) = \varepsilon_2^0 + zk_2 \quad \text{and} \quad \varepsilon_6(z) = \varepsilon_6^0 + zk_6 \qquad \dots(4.18)$$

Now from eq. 4.1, for distance z

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}_z = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}_z \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix}_z$$

$$= \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}_z \left\{ \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} + z \begin{Bmatrix} k_1 \\ k_2 \\ k_6 \end{Bmatrix} \right\} \qquad \dots(4.19)$$

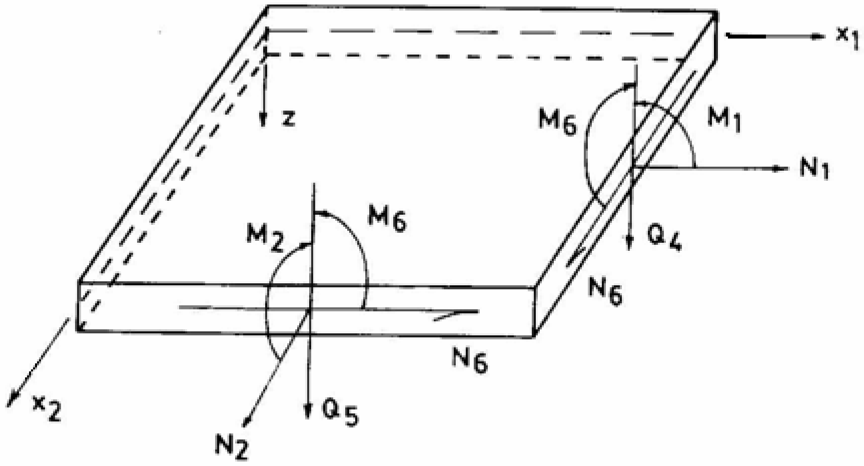


Fig 4.3 Stress and moment resultants

The stress and moment resultants (fig. 4.3) are evaluated per unit length of the laminate as follows:

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} dz \quad \text{and} \quad \begin{Bmatrix} M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} z\, dz \qquad \textbf{... (4.20)}$$

Thus,

$$N_1 = \int_{-h/2}^{h/2} \sigma_1\, dz = \int_{-h/2}^{h/2} [Q_{11} \quad Q_{12} \quad Q_{16}] \left\{ \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} + z \begin{Bmatrix} k_1 \\ k_2 \\ k_6 \end{Bmatrix} \right\} dz$$

$$= [A_{11} \quad A_{12} \quad A_{16}] \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} + [B_{11} \quad B_{12} \quad B_{16}] \begin{Bmatrix} k_1 \\ k_2 \\ k_6 \end{Bmatrix} \qquad \textbf{... (4.21)}$$

where $A = \int_{-h/2}^{h/2} Q\, dz$ and $B = \int_{-h/2}^{h/2} Q\, z dz$

$$M_1 = \int_{-h/2}^{h/2} \sigma_1 z\, dz = \int_{-h/2}^{h/2} [Q_{11} \quad Q_{12} \quad Q_{16}] \left\{ \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} + z \begin{Bmatrix} k_1 \\ k_2 \\ k_6 \end{Bmatrix} \right\} dz$$

$$= [B_{11} \quad B_{12} \quad B_{16}] \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} + [D_{11} \quad D_{12} \quad D_{16}] \begin{Bmatrix} k_1 \\ k_2 \\ k_6 \end{Bmatrix} \qquad ...(\mathbf{4.22})$$

where $B = \int_{-h/2}^{h/2} Q\, z dz$ and $D = \int_{-h/2}^{h/2} Q\, z^2 dz$

All stress and moment resultants can also be expressed as

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \\ M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \\ k_1 \\ k_2 \\ k_6 \end{Bmatrix} \qquad \textbf{… (4.23)}$$

$$\text{with } (A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} Q_{ij}\,(1, z, z^2)\,dz; \qquad i, j = 1, 2, 6 \qquad \textbf{… (4.24)}$$

Equation 4.23 represents the constitutive relations for a general laminate, and A_{ij}, B_{ij}, and D_{ij} are the inplane, extension bending coupling and bending stiffnesses, respectively. All these stiffnesses are derived for a unit length of the laminate. The elastic properties of each lamina are generally assumed to be constant through its thickness, as these laminae are considered to be thin. Then A_{ij}, B_{ij}, and D_{ij} are approximated as

$$A_{ij} = \sum_{k=1}^{n} (Q_{ij})_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} (Q_{ij})_k (z_k^2 - z_{k-1}^2) \qquad \textbf{… (4.25)}$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} (Q_{ij})_k (z_k^3 - z_{k-1}^3)$$

Several types of mechanical coupling in a general laminate are shown in eq. 4.23.These are grouped together as follows:

Extension-Shear	:	A_{16}, A_{26}
Extension-Bending	:	B_{11}, B_{12}, B_{22}
Extension -Twisting	:	B_{16} , B_{26}
Shear-Bending	:	B_{16} , B_{26}
Shear-Twisting	:	B_{66}
Bending-Twisting	:	D_{16} , D_{26}
Biaxial-Extension	:	A_{12}
Biaxial-Bending	:	D_{12}

Chapter 5

BENDING AND VIBRATION OF LAMINATED PLATES

BENDING AND VIBRATION OF LAMINATED PLATES

5.1 INTRODUCTION

The formulae presented in this chapter are based on the classical bending theory of thin composite plates. The small deflection bending theory for a thin laminate composite plate is developed based on Bernoulli's assumptions for bending of an isotropic thin plate. The development of the classical bending theory for a thin laminated composite plate follows Kirchhoff's assumptions for the bending of an isotropic plate. Kirchhoff's main suppositions are as follows [13]:

- The material behavior is linear and elastic.
- The plate is initially flat.
- The thickness of the plate is small compared to other dimensions.
- The translational displacements (u_1^0, u_2^0 and w) are small compared to the plate thickness, and the rotational displacements ($u_{1,1}^0$ and $u_{2,2}^0$) are very small compared to unity.
- The normals to the undeformed middle plane are assumed to remain straight, normal and inextensional during the deformation so that transverse normal and shear strains are neglected in deriving the plate kinematic relations.

5.2 THIN LAMINATED PLATE THEORY

Consider, a rectangular, thin laminated composite plate of length a, width b and thickness h as shown in fig.5.1. The plate consists of a laminate having n number of laminae with different materials, fibre orientations and thicknesses. The plate is subjected to surface loads q's and m's per unit area of the plate as well as edge loads $\bar{N}_1, \bar{N}_2$ and $\bar{N}_6$ per unit length. Let us consider u_1^0, u_2^0 and w are mid-plane displacement components. It is assumed that Kirchhoff's assumptions for the small deflection bending theory of a thin plate are valid in the present case. One of these assumptions is related to transverse strains $\varepsilon_3 \ (=\gamma_{zz}), \varepsilon_4 \ (=\gamma_{2z})$ and $\varepsilon_5 \ (=\gamma_{1z})$ which are neglected in derivation of plate kinematic relations i.e., stress-strain relations [13].

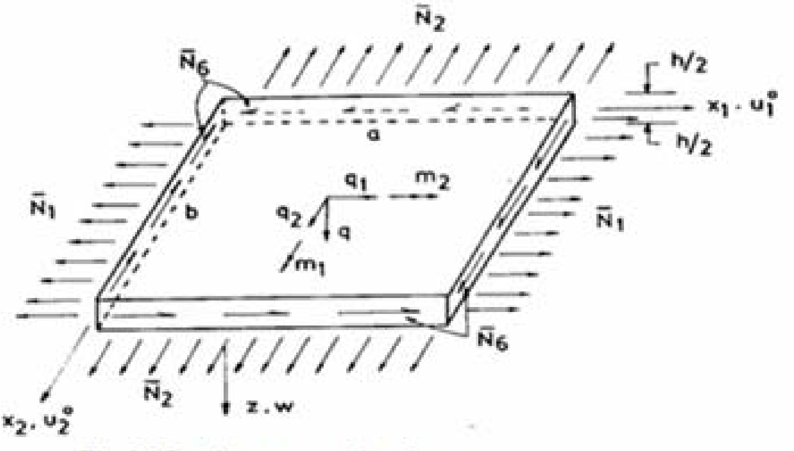


Fig 5.1 Laminate composite plate

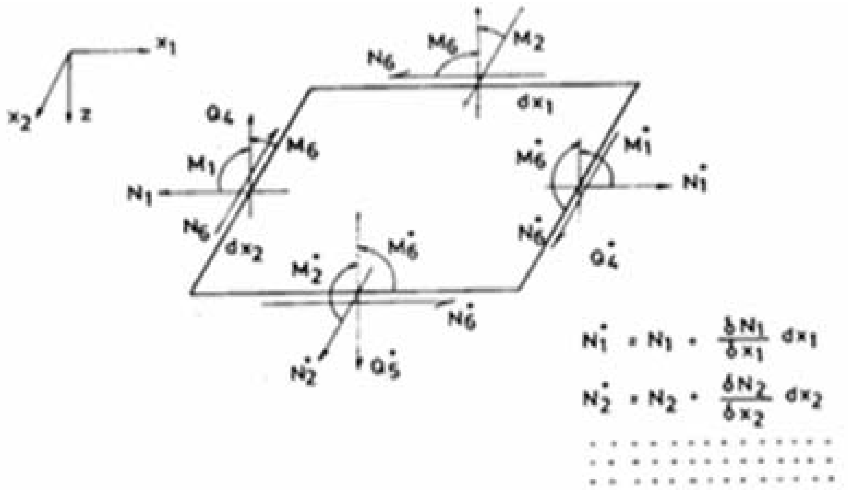


Fig 5.2 Force and moment resultants

Considering the dynamic equilibrium of an infinitesimally small element dx_1 dx_2 (fig. 5.2), the following equations of motion are obtained

$$N_{1,1} + N_{6,2} + q_1 = 0 \quad \textbf{... (5.1)}$$

$$N_{6,1} + N_{2,2} + q_2 = 0 \quad \textbf{... (5.2)}$$

$$Q_{4,1} + Q_{5,2} + q + \bar{N}_1 w_{,11} + \bar{N}_2 w_{,22} + 2\bar{N}_6 w_{,12} = 0 \quad \textbf{... (5.3)}$$

$$M_{1,1} + M_{6,2} = Q_4 - m_1 \quad \textbf{... (5.4)}$$

$$M_{2,2} + M_{6,1} = Q_5 - m_2 \quad \textbf{... (5.5)}$$

where commas are used to denote partial differentiation. Combining equations (5.3) through (5.5), we obtain

$$M_{1,11} + 2M_{6,12} + M_{2,22} + q + \bar{N}_1 w_{,11} + \bar{N}_2 w_{,22} + 2\bar{N}_6 w_{,12}$$
$$+ m_{1,1} + m_{2,2} = 0 \quad \textbf{... (5.6)}$$

Substituting equation (4.23) in equations (5.1), (5.2) and (5.6) and noting equations (4.16) and (4.17), we obtain the following governing differential equations in terms of mid-plane displacements u_1^0, u_2^0 and w.

$$A_{11} u^0_{1,11} + 2A_{16} u^0_{1,12} + A_{66} u^0_{1,22} + A_{16} u^0_{2,11} + (A_{12} + A_{66}) u^0_{2,12} + A_{26} u^0_{2,22}$$
$$- B_{11} w_{,111} - 3B_{16} w_{,112} - (B_{12} + 2B_{66}) w_{,122} - B_{26} w_{,222} + q_1 = 0 \quad \textbf{... (5.7)}$$

$$A_{16} u^0_{1,11} + (A_{12} + A_{66}) u^0_{1,12} + A_{26} u^0_{1,22} + A_{66} u^0_{2,11} + 2A_{26} u^0_{2,12} + A_{22} u^0_{2,22}$$
$$- B_{16} w_{,111} - (B_{12} + 2B_{66}) w_{,112} - 3B_{26} w_{,122} - B_{22} w_{,222} + q_2 = 0 \quad \textbf{... (5.8)}$$

$$D_{11} w_{,1111} + 4D_{16} w_{,1112} + 2(D_{12} + 2D_{66}) w_{,1122} + 4D_{26} w_{,1222} + D_{22} w_{,2222}$$
$$- B_{11} u^0_{1,111} - 3B_{16} u^0_{1,112} - (B_{12} + 2B_{66}) u^0_{1,122} - B_{26} u^0_{1,222} - B_{16} u^0_{2,111}$$
$$- (B_{12} + 2B_{66}) u^0_{2,112} - 3B_{26} u^0_{2,122} - B_{22} u^0_{2,222}$$
$$= q + m_{1,1} + m_{2,2} \bar{N}_1 w_{,11} + 2\bar{N}_6 w_{,12} + \bar{N}_2 w_{,22} \quad \textbf{... (5.9)}$$

Consider effect of transverse load q only on the laminates. Equations (5.7) to (5.9) reduce to

$$A_{11}u^0_{1,11} + 2A_{16}u^0_{1,12} + A_{66}u^0_{1,22} + A_{16}u^0_{2,11} + (A_{12}+A_{66})u^0_{2,12} + A_{26}u^0_{2,22}$$
$$-B_{11}w_{,111} - 3B_{16}w_{,112} - (B_{12}+2B_{66})w_{,122} - B_{26}w_{,222} = 0 \qquad \textbf{... (5.10)}$$

$$A_{16}u^0_{1,11} + (A_{12}+A_{66})u^0_{1,12} + A_{26}u^0_{1,22} + A_{66}u^0_{2,11} + 2A_{26}u^0_{2,12} + A_{22}u^0_{2,22}$$
$$-B_{16}w_{,111} - (B_{12}+2B_{66})w_{,112} - 3B_{26}w_{,122} - B_{22}w_{,222} = 0 \qquad \textbf{... (5.11)}$$

$$D_{11}w_{,1111} + 4D_{16}w_{,1112} + 2(D_{12}+2D_{66})w_{,1122} + 4D_{26}w_{,1222} + D_{22}w_{,2222}$$
$$-B_{11}u^0_{1,111} - 3B_{16}u^0_{1,112} - (B_{12}+2B_{66})u^0_{1,122} - B_{26}u^0_{1,222} - B_{16}u^0_{2,111}$$
$$-(B_{12}+2B_{66})u^0_{2,112} - 3B_{26}u^0_{2,122} - B_{22}u^0_{2,222} = q \qquad \textbf{... (5.12)}$$

Symmetric laminates:

For symmetric laminated plates, the extension-bending coupling matrix [B] = 0 and the above equations simplify to

$$A_{11}u^0_{1,11} + 2A_{16}u^0_{1,12} + A_{66}u^0_{1,22} + A_{16}u^0_{2,11} + (A_{12}+A_{66})u^0_{2,12} + A_{26}u^0_{2,22} = 0 \quad \textbf{... (5.13)}$$

$$A_{16}u^0_{1,11} + (A_{12}+A_{66})u^0_{1,12} + A_{26}u^0_{1,22} + A_{66}u^0_{2,11} + 2A_{26}u^0_{2,12} + A_{22}u^0_{2,22} = 0 \quad \textbf{... (5.14)}$$

$$D_{11}w_{,1111} + 4D_{16}w_{,1112} + 2(D_{12}+2D_{66})w_{,1122} + 4D_{26}w_{,1222} + D_{22}w_{,2222} = q \quad \textbf{... (5.15)}$$

5.3 FREE VIBRATION OF LAMINATED PLATE

5.3.1 UNSYMMETRICAL CROSS-PLY LAMINATED PLATE

For cross ply laminates, the material directions are oriented at 0 or 90 degrees.

For the stacking sequence $[0/90]_n$ of the laminate

$A_{16} = A_{26} = B_{12} = B_{16} = B_{26} = B_{66} = D_{16} = D_{26} = 0$

$A_{11} = A_{22}$, $B_{22} = -B_{11}$, $D_{22} = D_{11}$

The static case of equations (5.13) to (5.15) for the present case are expressed as

$$A_{11}u^0_{1,11} + A_{66}u^0_{1,22} + (A_{12}+A_{66})u^0_{2,12} - B_{11}w_{,111} = 0 \qquad \textbf{... (5.16)}$$

$$(A_{12}+A_{66})u^0_{1,12} + A_{66}u^0_{2,11} + A_{22}u^0_{2,22} + B_{11}w_{,222} = 0 \qquad \textbf{... (5.17)}$$

$$D_{11}(w_{,1111} + w_{,2222}) + 2(D_{12}+2D_{66})w_{,1122} - B_{11}(u^0_{1,111} - u^0_{2,222}) = q \quad \textbf{... (5.18)}$$

The equation of motion is expressed as

$$A_{11}u^0_{1,11} + A_{66}u^0_{1,22} + (A_{12} + A_{66})u^0_{2,12} - B_{11}w_{,111} = 0 \quad \text{... (5.19)}$$

$$(A_{12} + A_{66})u^0_{1,12} + A_{66}u^0_{2,11} + A_{22}u^0_{2,22} + B_{11}w_{,222} = 0 \quad \text{... (5.20)}$$

$$D_{11}(w_{,1111} + w_{,2222}) + 2(D_{12} + 2D_{66})w_{,1122} - B_{11}(u^0_{1,111} - u^0_{2,222}) - \rho h\ddot{w} = 0 \quad \text{... (5.21)}$$

where ρ is the average density of laminate.

In the above equations, the transverse inertia is only included and the in plane inertia forces are neglected [15].

Boundary conditions for simply supported laminated plate

For $x_1 = 0, a$:

$$w = 0; \quad M_1 = B_{11}u^0_{1,1} - D_{11}w_{,11} - D_{12}w_{,22} = 0$$

$$u^0_2 = 0; \quad N_1 = A_{11}u^0_{1,1} + A_{12}u^0_{2,2} - B_{11}w_{,11} = 0$$

For $x_2 = 0, b$:

$$w = 0; \quad M_2 = -B_{11}u^0_{2,2} - D_{12}w_{,11} - D_{11}w_{,22} = 0$$

$$u^0_1 = 0; \quad N_2 = A_{12}u^0_{1,1} + A_{11}u^0_{2,2} - B_{11}w_{,22} = 0 \quad \text{... (5.22)}$$

In the equation (5.21), transverse inertia is only included.

Solution of the above equations (5.19), (5.20) and (5.21) which satisfy the boundary conditions given in the equation (5.22) is written as [15]

$$u^0_1 = A\cos\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b}e^{i\omega t}$$

$$u^0_2 = B\sin\frac{m\pi x_1}{a}\cos\frac{n\pi x_2}{b}e^{i\omega t}$$

$$w = C\sin\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b}e^{i\omega t} \quad \text{... (5.23)}$$

Substituting (5.23) into above equations (5.19), (5.20) and (5.21) yield the following

homogeneous algebraic equations

$$\begin{bmatrix} A_{mn} & B_{mn} & C_{mn} \\ B_{mn} & D_{mn} & E_{mn} \\ C_{mn} & E_{mn} & (F_{mn} - \lambda) \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \text{... (5.24)}$$

where

$$A_{mn} = A_{11} m^2 + A_{66} n^2 k^2$$

$$B_{mn} = (A_{12} + A_{66}) mnk$$

$$C_{mn} = -B_{11} \frac{m^3 \pi}{kb}$$

$$D_{mn} = A_{66} m^2 + A_{11} n^2 k^2$$

$$E_{mn} = \frac{B_{11} \pi n^3 k^2}{b}$$

$$F_{mn} = \frac{\pi^2}{k^2 b^2} \left[D_{11} (m^4 + n^4 k^4) + 2(D_{12} + 2 D_{66}) m^2 n^2 k^2 \right]$$

$$\lambda = \frac{\rho h \omega^2 k^2 b^2}{\pi^2} \quad \text{and} \quad k = \frac{a}{b}$$

where k is aspect ratio. a and b are plate dimensions in x_1 and x_2 directions.

In order to obtain a nontrivial solution to (5.24) the determinant of the coefficient matrix must vanish. Thus

$$\begin{vmatrix} A_{mn} & B_{mn} & C_{mn} \\ B_{mn} & D_{mn} & E_{mn} \\ C_{mn} & E_{mn} & (F_{mn} - \lambda) \end{vmatrix} = 0 \qquad \textbf{... (5.25)}$$

Expanding the determinant in the above equation (5.25), the circular frequency is obtained.

$$\omega_{mn}^2 = \frac{\pi^4}{(\rho h k^4 b^4)} \left[D_{11} (m^4 + n^4 k^4) + 2(D_{12} + 2 D_{66}) m^2 n^2 k^2 - \frac{B_{11}^2}{J_3} (m^4 J_1 + n^4 k^4 J_2) \right] \qquad \textbf{... (5.26)}$$

where

$$J_1 = A_{66} m^4 + A_{11} m^2 n^2 k^2 + (A_{12} + A_{66}) n^4 k^4$$

$$J_2 = A_{66} n^4 k^4 + A_{11} m^2 n^2 k^2 + (A_{12} + A_{66}) m^4$$

$$J_3 = \left[(A_{66} n^2 k^2 + A_{11} m^2)(A_{66} m^2 + A_{11} n^2 k^2) - (A_{12} + A_{66})^2 m^2 n^2 k^2 \right] \qquad \textbf{... (5.27)}$$

5.3.2 SIMPLY SUPPORTED SYMMETRICAL CROSS-PLY LAMINATED PLATE

For symmetric laminated plates, [B] = 0. The circular frequency is expressed as

$$\omega_{mn}^2 = \frac{\pi^4}{(\rho h k^4 b^4)} \left[D_{11} (m^4 + n^4 k^4) + 2(D_{12} + 2 D_{66}) m^2 n^2 k^2 \right] \qquad \textbf{... (5.28)}$$

5.3.3 SIMPLY SUPPORTED SYMMETRICAL SPECIALLY ORTHOTROPIC SANDWICH PLATE

For specially orthotropic plates, $A_{16}= A_{26}= D_{16}= D_{26}=0$ and $[B] = 0$.

From above equations (5.15) and (5.21), the circular frequency of the simply supported sandwich plate is expressed as [10]

$$\omega_{mn}^{2} = \frac{\pi^4}{(\rho h k^4 b^4)}\left[D_{11} m^4 + 2(D_{12} + 2D_{66})m^2 n^2 k^2 + D_{22} n^4 k^4\right] \quad \textbf{... (5.29)}$$

and

$$\omega_{mn} = \left(\frac{\pi^2}{a^2}\right)\left(\frac{D_{11}}{\rho h}\right)^{(1/2)}\left[m^4 + \frac{2(D_{12}+2D_{66})m^2 n^2 k^2}{D_{11}} + \frac{D_{22} n^4 k^4}{D_{11}}\right]^{(1/2)} \quad \textbf{... (5.30)}$$

The natural frequency of the sandwich plate is expressed as

$$\nu_{mn} = \left(\frac{\pi}{2a^2}\right)\left(\frac{D_{11}}{\rho h}\right)^{(1/2)}\left[m^4 + \frac{2(D_{12}+2D_{66})m^2 n^2 k^2}{D_{11}} + \frac{D_{22} n^4 k^4}{D_{11}}\right]^{(1/2)} \quad \textbf{...(5.31)}$$

The nondimensional frequency parameter ($\lambda_{o(mn)}$) is given by equation (5.32)

$$\begin{aligned}\lambda_{o(mn)} &= \nu_{mn} a^2 \left(\frac{\rho h}{D_{11}}\right)^{(1/2)} \\ &= \left(\frac{\pi}{2}\right)\left[m^4 + \frac{2(D_{12}+2D_{66})m^2 n^2 k^2}{D_{11}} + \frac{D_{22} n^4 k^4}{D_{11}}\right]^{(1/2)} \quad \textbf{... (5.32)}\end{aligned}$$

$$\lambda = \frac{\rho h \omega^2 k^2 b^2}{\pi^2} \quad \text{and} \quad k = \frac{a}{b}$$

5.3.4 SIMPLY SUPPORTED ISOTROPIC PLATE

In case of isotropic plate, natural frequency of the isotropic plate is given by

$$\nu_{mn} = \left(\frac{\pi}{2a^2}\right)\left(\frac{D}{\rho h}\right)^{(1/2)}\left[m^2 + n^2 k^2\right] \quad \textbf{... (5.33)}$$

The nondimensional frequency parameter ($\lambda_{o(mn)}$) is given by equation (5.34)

$$\lambda_{o(mn)} = \nu_{mn} a^2 \left(\frac{\rho h}{D}\right)^{(1/2)} = \left(\frac{\pi}{2}\right)\left[m^2 + n^2 k^2\right] \quad \textbf{... (5.34)}$$

where D = Flexural rigidity of the plate.

Chapter 6

SANDWICH STRUCTURE

SANDWICH STRUCTURE

6.1 SANDWICH PRINCIPLE

The basic prerequisite for high-performance structural component parts as used in aerospace applications is light-weight design wherever possible. An essential component of these light-weight structures is load-bearing and buckling optimized shell elements. The classical method to obtain improved buckling properties is using sandwich structures have also proven their worth in a number of fields. The performance of a sandwich structure depends primarily upon the efficiency of surface skins and the distance between them. A great distance between the surface skins produces a correspondingly great geometrical moment of inertia, thus leading to high bending stiffness. Since this arrangement subjects the core of the sandwich to a relatively small amount of stress, it can be reduced in weight significantly. Extremely thin-walled sandwich structures present the problem of how force is introduced and the sandwich structure's sensitivity towards impact loads. This means that a minimum wall thickness is required for the surface skins to be able to ensure that it is adequate to the purpose.

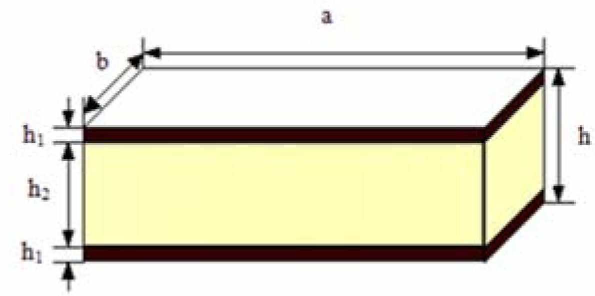


Fig 6.1 Typical Geometry of Sandwich Plate

Fig 6.2 Aluminium honeycomb core

6.1.1 FACE SHEETS Aluminium honeycomb core

The face sheets provide the flexural rigidity of the sandwich structure. It should also possess tensile and compressive strength.

6.1.2 CORES

The purpose of the core is to increase the flexural stiffness of the panel. The core in general has low density in order to add as little as possible to the total weight of the sandwich construction. The core must be stiff enough in shear and perpendicular to the faces to ensure that face sheets are distant apart. In addition the core must withstand compressive loads without failure.

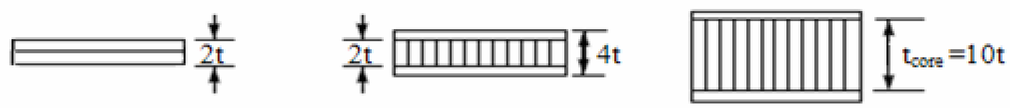
6.1.2.1 ALUMINUM HONEYCOMB

These cores are available in variety of materials for sandwich structures. These cores can be formed to any shape or curve without excessive heating or mechanical force. Honeycombs have very high stiffness perpendicular to the faces and the highest shear stiffness and strength to weight ratios of the available core materials. The most commonly used honeycombs are made of aluminum or impregnated glass or aramid fiber mats such as nomex and thermoplastic honeycombs.

6.2 STIFFNESS TO WEIGHT RATIO OF SANDWICH PLATE

Let us consider a stiffened single skin structure and sandwich structure as shown in figure 6.3. Three cases are considered for stiffness to weight ratio analysis.

Face thickness= t and Mass of Face =25xMass of Core



Total Thickness =2t — Case I

Total Thickness=4t — Case II

Total Thickness=12t — Case III

Fig 6.3 (Bending Stiffness / Weight Ratios) for Sandwich and Single Skin Cross Sections

Case I Total $Mass_1$ =2x$Mass_{face}$ and Relative Bending Stiffness=1

Case II Total Mass=1.04x Total $Mass_1$ and Relative Bending Stiffness=6.73

Case III Total Mass= 1.2x Total $Mass_1$ and Relative Bending Stiffness=75.8

.From Case III discussion, for a face/core thickness ratio =1/10, which gives a relative bending stiffness 75.8 times the stiffness of the equivalent single skin. Hence, sandwich is a structurally efficient structure with regard to stiffness/ weight ratio.

Chapter 7

RESULTS AND DISCUSSIONS OF ISOTROPIC PLATE

RESULTS AND DISCUSSIONS OF ISOTROPIC PLATE

7.1 EXPERIMENTAL RESULTS

In the present investigation, frequencies of vibration of rectangular plate with various support boundary conditions are studied by means of experiments. The results obtained using experiments are compared with Finite Element package (ANSYS) and theoretical calculations.

An isotropic Aluminium plate is of dimensions (49.5 cm x 36.1 cm) when measured by a meter scale and the average thickness was 2.75 mm when measured by micrometer screw gauge.

Table 7.1 Properties for Aluminium

Serial Number	Property	Value
1	Young's Modulus	70GPa
2	Poisson's Ratio	0.33
3	Density	2700 kg/m^3

The Test facility

The Test facility used to find the frequencies of vibration is shown in the fig (7.1). The setup consists mainly of three subsystems

i) The Loading System

ii) The Excitation System

iii) The Measuring System

The loading frame consists of two vertical steel I beams with two cross members, one at the top and the other at some height from the ground. The loading system facilitated the holding of the specimen and provided the necessary boundary conditions. The excitation system consisted of an oscillator, a power amplifier and an electro-dynamic shaker. A sinusoidal electrical signal was generated by the oscillator. The output of the oscillator was fed to the shaker through the power amplifier. The shaker was kept in alignment with the loading system and the shaker head was fastened to the threaded stud attached to the specimen by means of a coupling nut. The excitation system facilitated the application of a sinusoidal dynamic load of the desired amplitude and frequency. The amplitude of the forced oscillation was measured in terms of the shaker current indicated by an output ammeter on

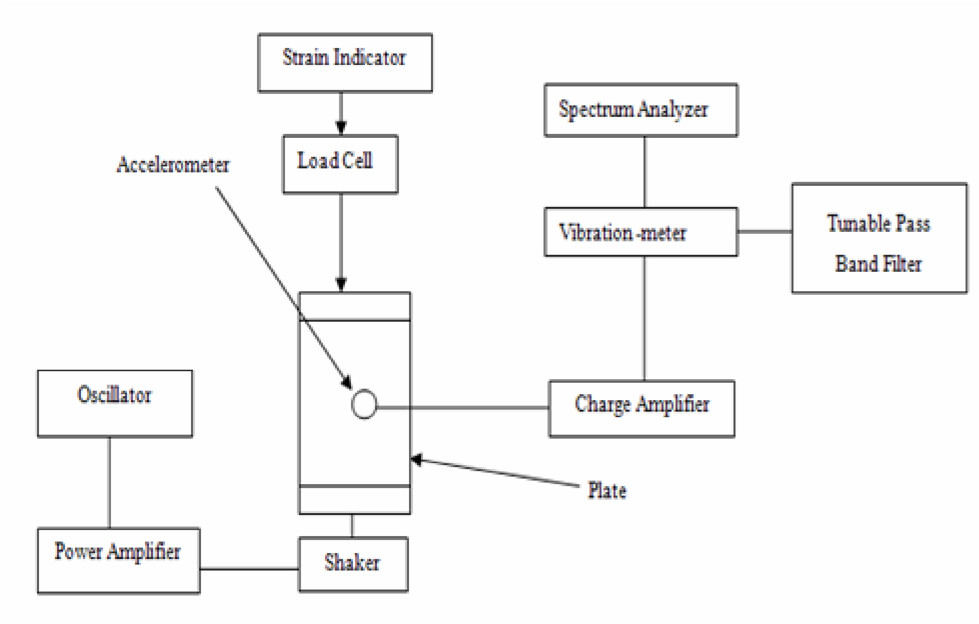


Fig 7.1 Schematic diagram for vibration experiment

Note: - Frequency means transverse vibration natural frequency

the power amplifier. A piezoelectric accelerometer was been stuck at a suitable location on the plate to pick up the plate response.

The Instrumentation:

The table gives information about the various instruments used for the experiment.

The Experimental Procedure

Every instrument used in the experiment was calibrated and adjusted. Then the experimental setup was organized as shown in the figures (7.1) and (7.2). The plate was attached to the loading system using the screw-jack mechanism and the adjusting screw. The threaded stud attached to the plate was then fastened to the shaker head by means of a coupling nut. During all these initial adjustments, the shaker was kept in standby mode. Also the amplitude knobs on the oscillator and on the power amplifier were kept at zero position.

The control was then switched to the operate position. The amplitude knob on the amplifier was turned and kept at a certain finite amplitude level. The desired amplitude level was obtained by adjusting the amplitude knob on the oscillator.

Then the frequency of excitation was slowly increased from a minimum value of 5 hertz. The response of the plate was constantly monitored on the vibration meter and the FFT analyzer.

As the frequency was increased, the response of the plate also increased and the frequency was kept constant when the plate was vibrating with the largest amplitude. This was the first significant resonance behavior and it corresponded to the first fundamental

Table 7.2 Specification of Instruments

S. No.	Instrument	Model	Manufacturer
1	Electrodynamic Shaker	Model-3520	Industrial Electronics
2	Power Amplifier	Model-3520	Industrial Electronics
3	Oscillator	Model 200 CD	Hewlett Packard
4	Charge Amplifier	Type 2635	Bruel and Kjaer
5	Tunable Band Pass Filter	Type 1621	Bruel and Kjaer
6	Spectrum Analyzer	SI-1220	Schlumberger
7	Oscilloscope	54603 B	Hewlett Packard
8	Piezoelectric Accelerometer	Type 4374	Bruel and Kjaer
9	Conditioning Amplifier	Type 2626	Bruel and Kjaer

frequency was kept constant when the plate was vibrating with the largest amplitude. This was the first significant resonance behavior and it corresponded to the first fundamental frequency, i.e. the first mode of vibration. The frequency of the plate was then equal to the excitation frequency. This is a characteristic resonance. As the frequency was further increase, again large amplitude plate vibrations were observed. This corresponded to the second mode of resonance. The corresponding excitation frequency was recorded. The procedure was repeated for further values of resonant frequencies corresponding to higher modes.

Results for different boundary conditions

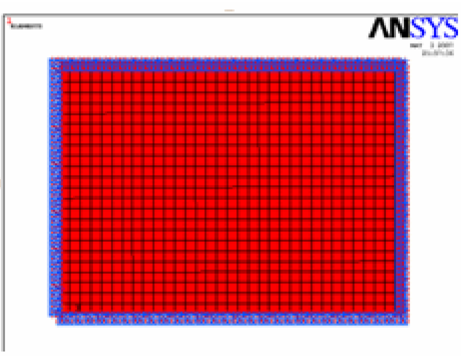
Table 7.3 Experimental results for clamped and simply supported

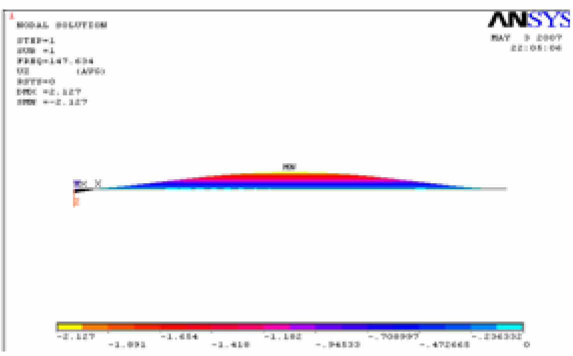
Experiment		
Mode	All Sides Clamped	All Sides Simply Supported
1		
2	240	154
3	373	229
4	414	287
5	449	302

7.2 FINITE ELEMENT PACKAGE (ANSYS) RESULTS

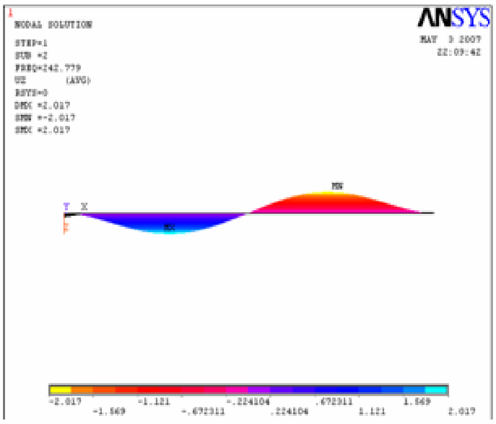
The modal analysis of isotropic plate is performed using the Finite Element Package (ANSYS). The inputs are considered as given in table 7.1. 792 elements are used for the present analysis. The mode shapes, deflections and frequencies are given below. SHELL 99 element is used for this analysis.

7.2.1 CLAMPED PLATE

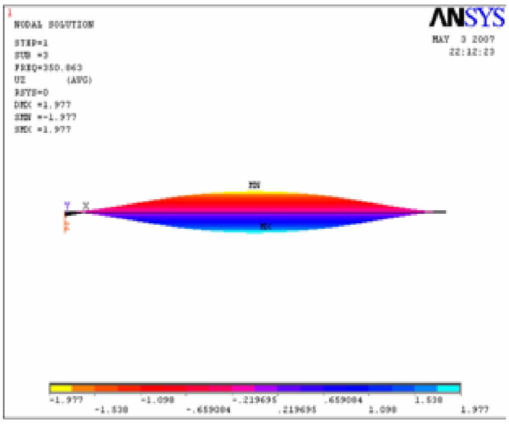




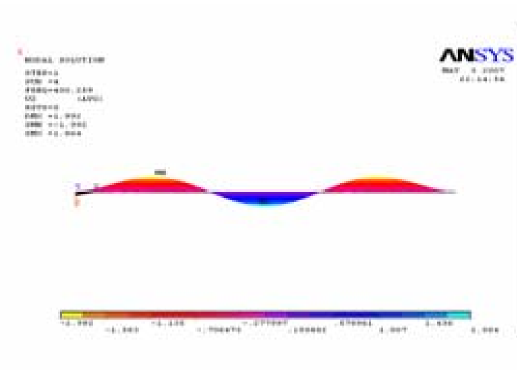
Meshed model First Mode



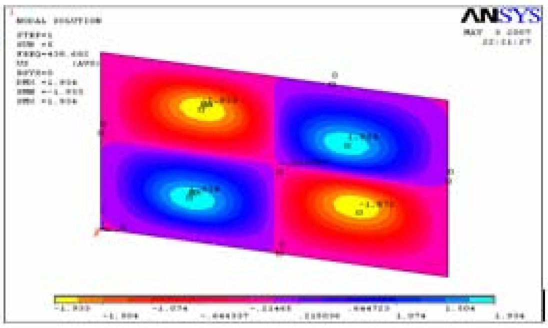
Second Mode



Third Mode



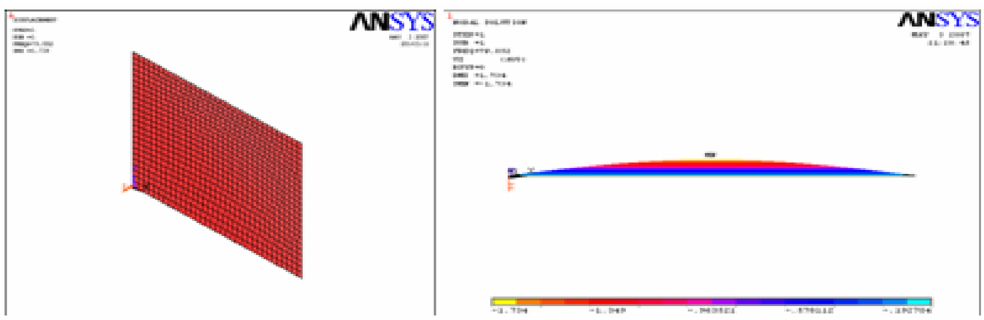
Fourth Mode



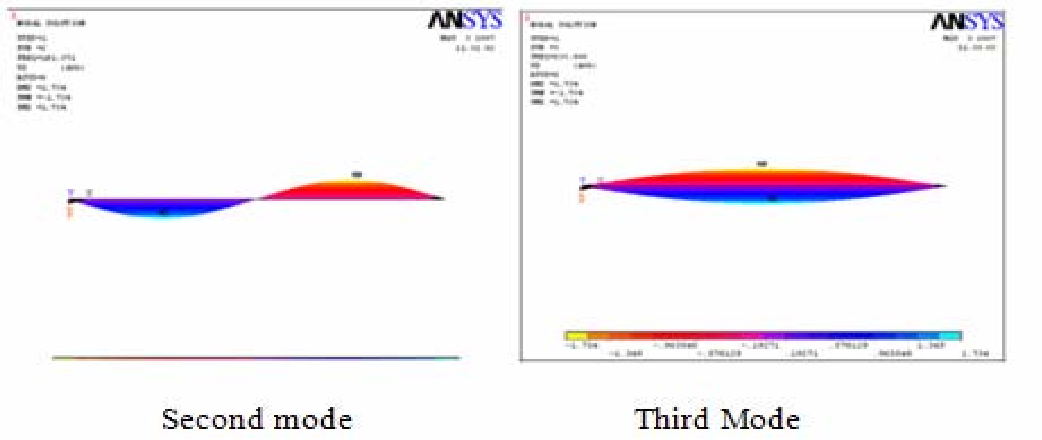
Fifth Mode

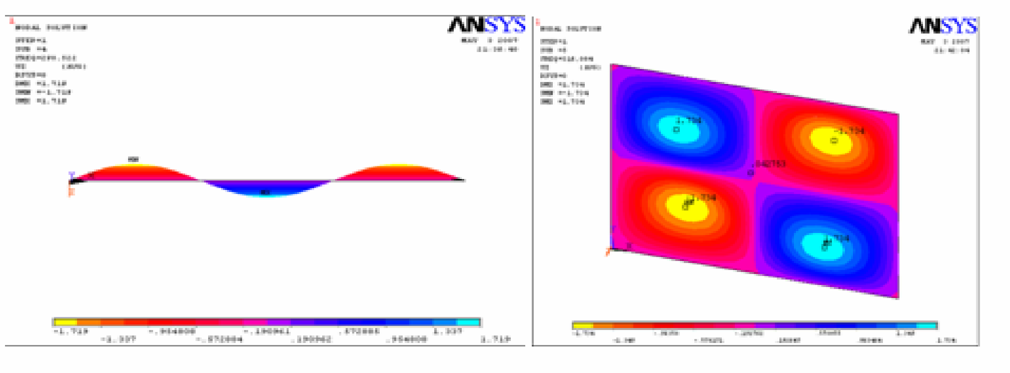
Fig7.2 Mode shapes for clamped plate

7.2.2 SIMPLY SUPPORTED PLATE



First Mode





Fourth Mode Fifth Mode

Fig 7.3 Mode shapes for simply supported plate

7.3 COMPARISON OF EXPERIMENTAL WORK AND FEA WORK WITH ANALYTICAL SOLUTION

The results of experimental, FEA and analytical work are comparable.

Table 7.4 Comparison of frequency (Hz) results of isotropic plate

Boundary Conditions	Mode Number	FEA(ANSYS) Results(Hz)	Experimental Results(Hz)	Analytical Results(Hz)	Error** (%)
c-c-c-c	1	147.63		148.17	
	2	242.63	240	243.21	+1.08
	3	350.86	373	352.62	-9.12
	4	400.26	414	402.56	-3.43
	5	438.68	449	443.92	-2.35
s-s-s-s	1	79.02		79.05	
	2	161.37	154	161.40	4.56
	3	233.81	229	233.87	2.05
	4	298.52	287	298.63	3.85
	5	316.08	302	316.23	4.45

** Errors are calculated by comparison of FEA (ANSYS) and experimental results.

Variation of frequency values in the analysis in comparison is due to consideration of assumptions (setup assumption).

Note:-S-Simply Supported and C- Clamped (Fixed)

7.4 EFFECT OF ASPECT RATIO ON NON-DIMENSIONAL FREQUENCY OF ISOTROPIC PLATE

Keeping all other parameters constant, the effect of aspect ratio on the natural frequency is analyzed. The nondimensional frequency parameter is given by equation (5.34) for simply support condition. The table below gives the natural frequencies of the aluminum plate for different values of aspect ratios. It also shows the associated mode shapes in brackets as (m, n).

Table 7.5 Non-Dimensional Frequency Parameter vs. Aspect Ratio

Aspect Ratio	Non dimensional Frequency Parameter				
	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)
0.2	1.63	1.82	2.13	2.57	3.14
0.5	1.96	3.14	5.1	7.85	11.38
1	3.14	7.85	15.7	26.69	40.82
1.5	5.1	15.7	33.36	58.09	89.88
2	7.85	26.69	58.09	102.05	158.57
3	15.7	58.09	128.74	227.65	354.82

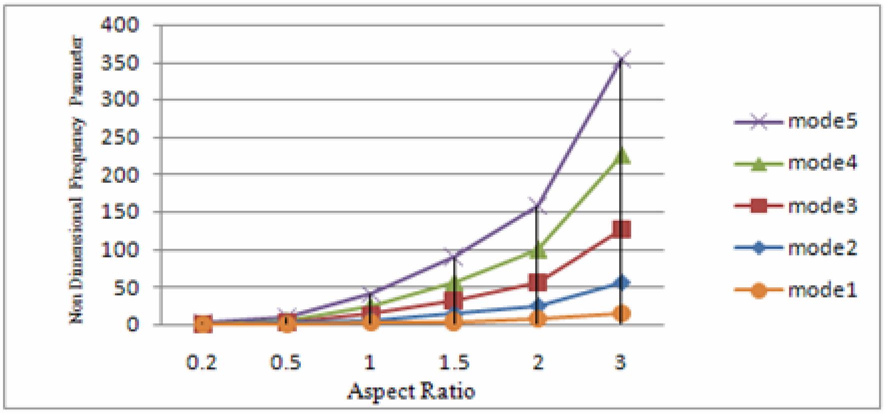


Fig 7.4 The graph shows that Non –Dimensional Frequency Parameter increases upon increase in Aspect Ratio

7.5 EFFECT OF CHANGE OF DIMENSION ON FREQUENCY (MASS CONSTANT) OF ISOTROPIC PLATE

The effect of change in dimension (width and thickness) of plate on the natural frequency is analyzed as

$$(\rho * a)\, b * t = \text{mass} = \text{constant}$$

Three cases are studied and results depend upon increment and decrement of width and thickness. 625 elements are used for the effect of dimension change on natural frequency. For the following cases the results are given in the table 7.3.

Case I $b_1 = b$ **and** $t_1 = t$

Case II $b_2 = b/2$ **and** $t_2 = 2t$

Case III $b_3 = b/3$ **and** $t_3 = 3t$

Table 7.6 Frequencies due to change in dimensions

S. No.	Mode Number	FEA(ANSYS) Results(Hz)		
		Case I	Case II	Case III
1	1	79.02	467.08	1465.9
2	2	161.37	631.19	1709.1
3	3	233.81	904.33	2113.2
4	4	298.52	1286.0	2676.3
5	5	316.08	1696.9	3153.5

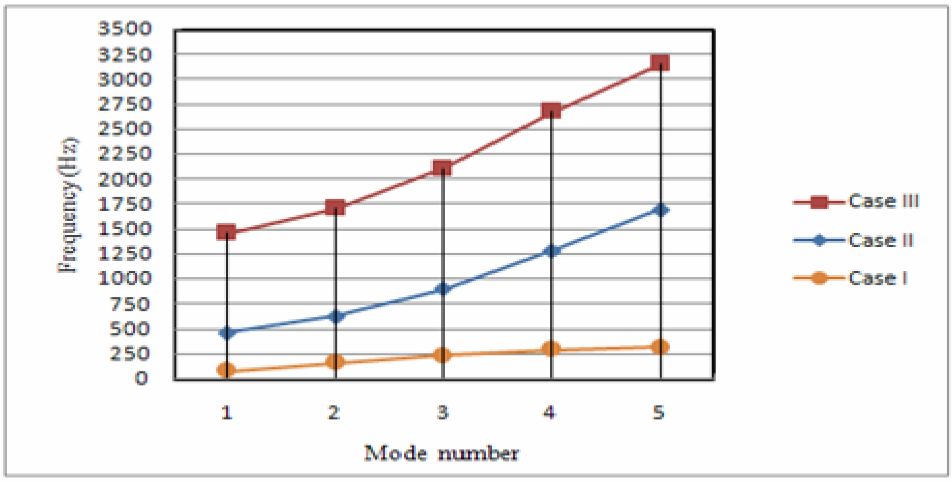


Fig 7.5 The graph shows that Frequencies increases gradually due to decrease in width and increase in thickness

Chapter 8

RESULTS AND DISCUSSIONS OF SANDWICH PANAL

RESULTS AND DISCUSSIONS OF SANDWICH PANAL

In this analysis the specimen is tested for simply supported condition and the results are generated which are compared with different methods. Material properties of honeycomb sandwich panel having aluminium face sheets are

Plate Description

Plate Dimensions:	1.828 * 1.292m (a*b)		
Thickness:	Core	6.35 mm	
	Face- sheets	0.4064 mm	
Material Properties			
	Core	G_{13}=0.1344 G Pa	G_{23}=0.0517 G Pa
		ν_c= .33	
	Face-Sheets	E=68.948 G Pa	
		ν_{12}= .32	
Mass-Density (Kg/m^3)	Core	121.79	
	Face-Sheets	2768	

8.1 EXPERIMENTAL RESULTS

In the present investigation, frequencies of vibration of sandwich panel with simple support condition are studied by means of experiment. The model is shown in fig 8.1.The results obtained using experiments are compared with Finite Element Package (ANSYS) and theoretical calculations. The table 8.1 gives information about the various instruments used for the experiment.

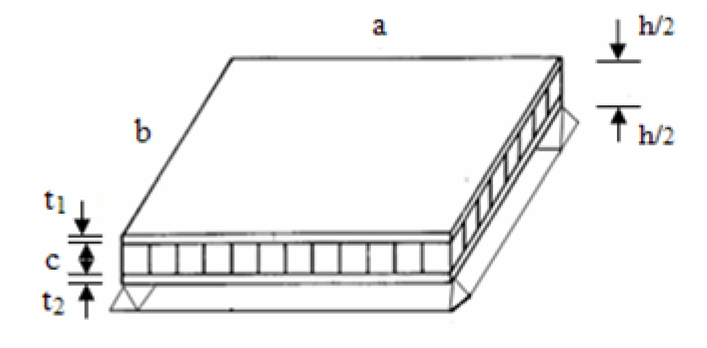


Fig 8.1 Three layered simply supported sandwich panel

The experimental procedure of vibration experiment is same as for the isotropic plate. The procedure is repeated for further values of resonant frequencies corresponding to higher modes. The experimental results are given in the 8.3 section.

Table 8.1 Specification of Instruments

S. No.	Instrument	Model
1	Electrodynamic Shaker	SEV050
2	Modular Power Amplifier	DSA 4K4
3	Digital Shaker Controller	DSP 2105 F
4	4 Channel Signal Conditioner	SPSC-04
5	Piezoelectric Accelerometer (PCB-USA)	357B21
6	Swept Sine Vibration Controller (Saraswati Wine Sine Software)	SVC 2205-F

8.2 FINITE ELEMENT PACKAGE (ANSYS) RESULTS

8.2.1 MODAL ANALYSIS

The modal analysis of sandwich panel is performed using the Finite Element Package (ANSYS). The inputs are considered as given in the experimental work. SHELL 99 (quadratic element) is used for the vibration analysis. 759 elements and 2390 nodes are used for accurate analysis. The mode shapes, deflections and frequencies are given below.

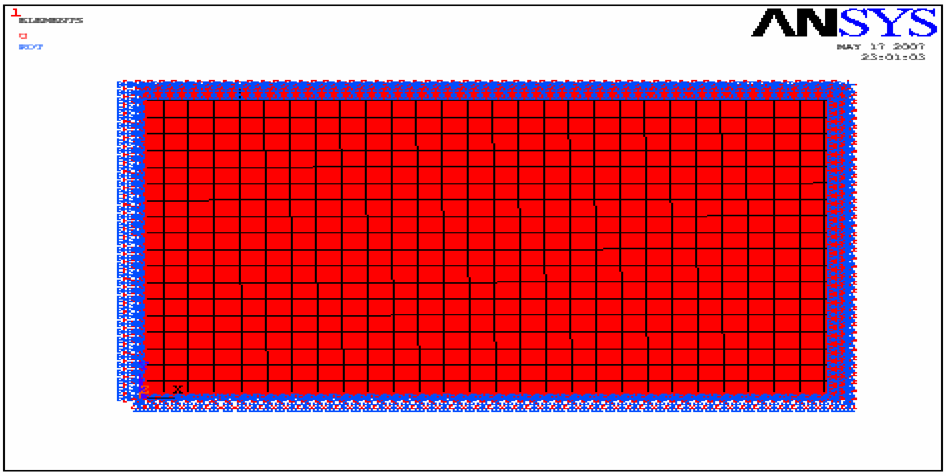


Fig 8.2 Meshed sandwich panel

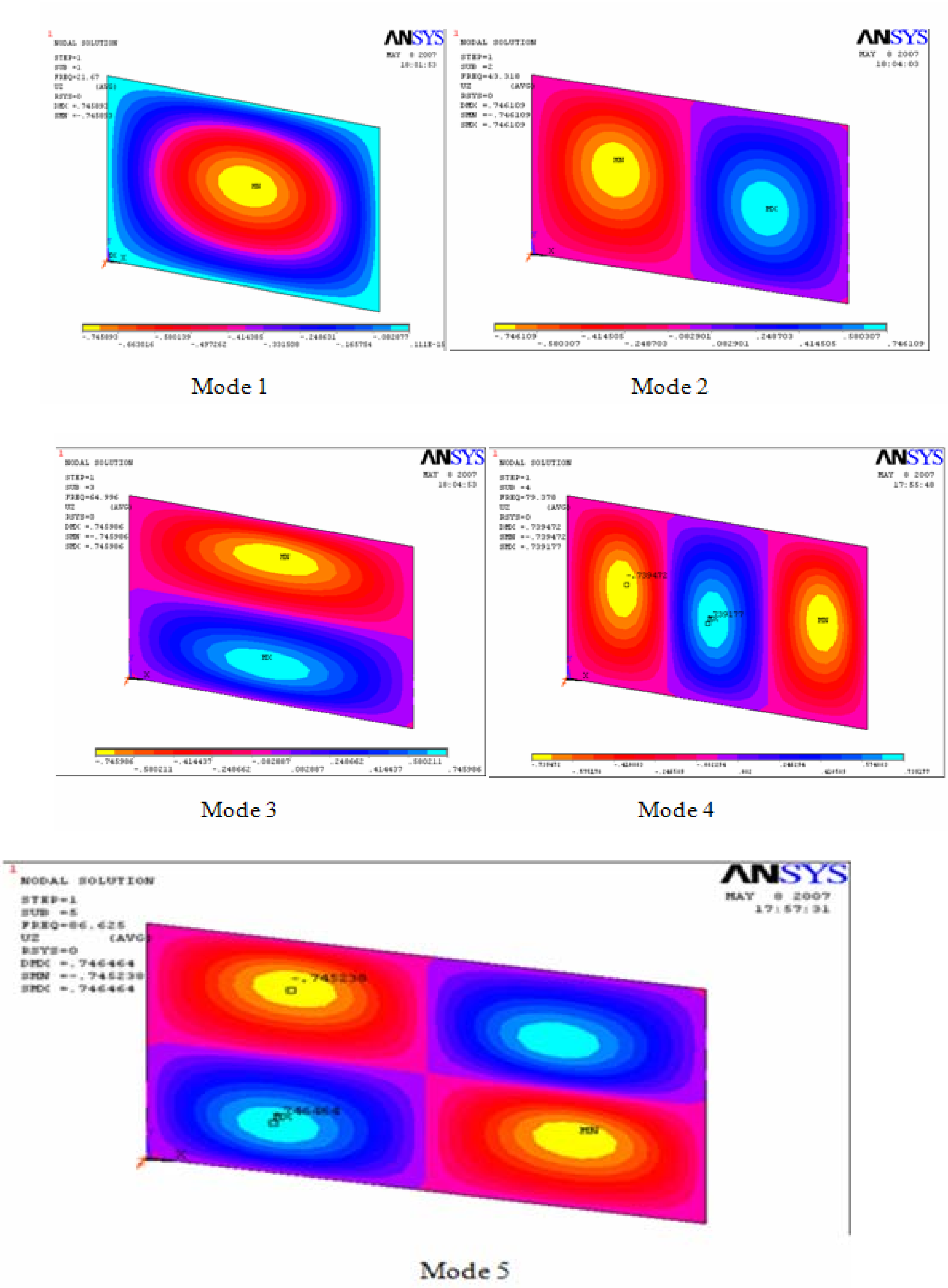


Fig 8.3 Mode shapes of simply supported sandwich panel

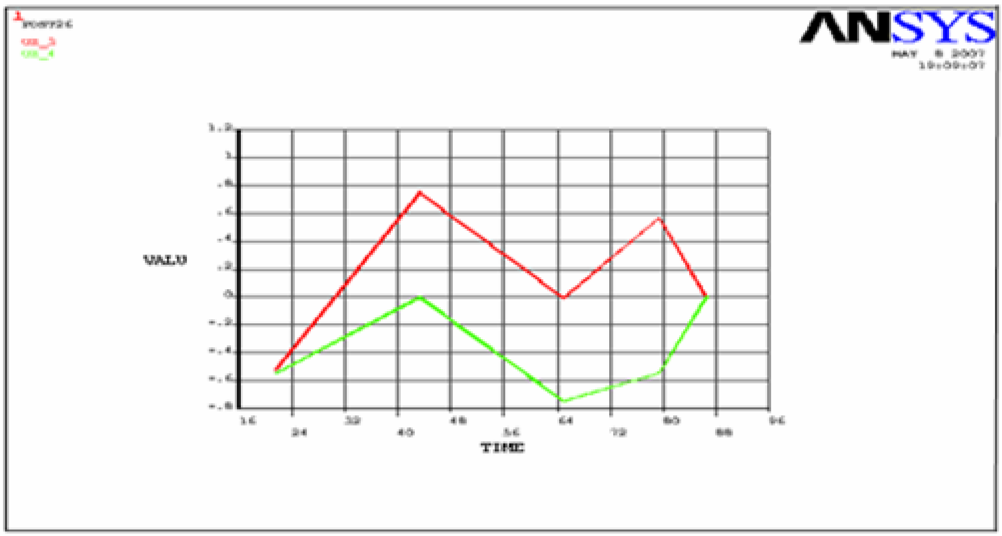


Fig 8.4 Deflection vs. frequency graph at nodes 1877 and 1333

8.2.2 CONVERGENCE STUDY

The results for different discretization of the simply supported sandwich plate are presented in Table 8.2. ANSYS uses 1x1, 2x2, 3x3 and 4x4 mesh sizes and corresponding elements 1218, 759, 532 and 294 for convergence study. The uniformly converging results assure the accuracy and correctness of the present analysis.

Table 8.2 Convergence Study of Frequency

S. No.	Mesh Size	Frequency (Hz)				
		Mode1	Mode2	Mode3	Mode4	Mode5
1	1x1	21.751	43.480	65.241	79.677	86.954
2	2x2	21.670	43.318	64.996	79.378	86.625
3	3x3	21.669	43.315	64.991	79.370	86.616
4	4x4	21.668	43.309	64.982	79.350	86.594

Note:- Frequency means transverse vibration natural frequency

8.3 COMPARISON OF FEA (ANSYS) WITH ANALYTICAL SOLUTION, EXPERIMENTAL WORK AND FEA (MSC/NASTRAN)

The results of the experiment presented in table 8.3 are good agreement with FEA and analytical analysis. Error calculated on the basis of comparison between results of FEA (ANSYS) and experimental values. The difference of results between these analyses is due to difference between modeling of sandwich panel and consideration of assumptions of material properties and setup arrangements.

Table 8.3 Comparison of experimental results with FEA (ANSYS) results, analytical results and FEA (MSC/NASTRAN)

	Frequency				
Mode Number	Experimental Results(Hz)	FEA (ANSYS) Results(Hz)	Analytical Results(Hz)	FEA (MSC/NASTRAN) Results(Hz)	Error** (%)
1		21.670	24.280	25.079	
2	44	43.315	48.248	51.396	+1.56
3	67	64.991	69.781	72.435	+3.08
4	78	79.370	82.454	81.974	-1.72
5	89	86.616	93.312	91.001	+2.75

** Error is calculated on the basis of comparison between FEA (ANSYS) and Experimental Results

Note: - Frequency means natural frequency of transverse vibration

8.4 STUDY OF COMPARISON BETWEEN SANDWICH PLATE AND EQUIVALENT PLATE

Comparison of modal analysis between simply supported sandwich plate and same dimensions aluminium face plate is shown in the table 8.4. Mesh sizes for present study are 1x1 and 2x2. Figure 8.3 shows that sandwich plate having 1.4 times higher fundamental frequency than equivalent plate. Increase in frequency is due increase in flexural stiffness of the plate.

Table 8.4 Comparison of Frequency between Sandwich Plate and Equivalent Plate

Mesh Size	Mode	Frequency (Hz)	
		Sandwich Plate	Equivalent Plate
1x1	1	21.751	15.426
	2	43.480	30.839
	3	65.241	46.278
	4	79.677	56.523
	5	86.954	61.687
2x2	1	21.670	15.425
	2	43.318	30.838
	3	64.996	46.277
	4	79.378	56.521
	5	86.625	61.685

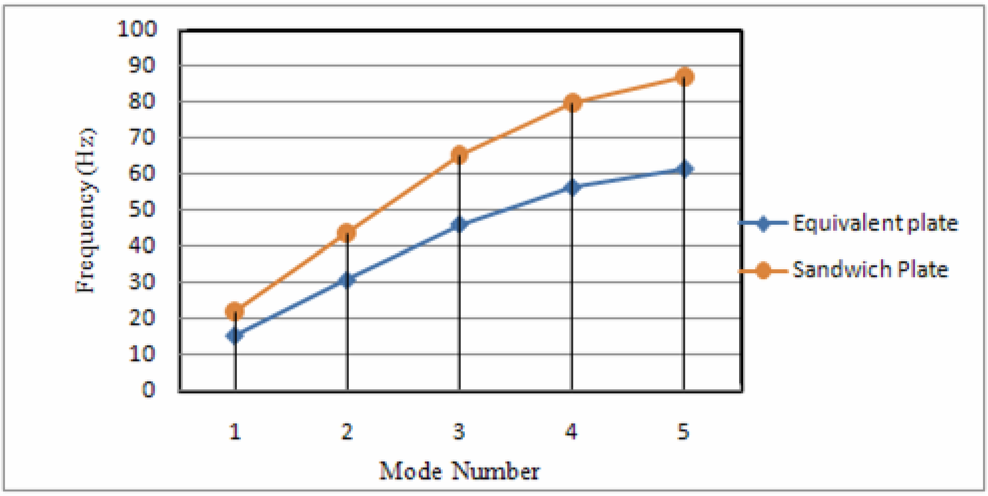


Fig 8.5 Variation of frequency between sandwich panel and equivalent panel

8.5 PARAMETER STUDIES USING FE MODEL

FE model (fig 8.2) is used to investigate the effects of various design parameters on the frequencies of free vibration of simply supported honeycomb sandwich plate.

8.5.1 EFFECT OF INCREASE IN THICKNESS OF THE CORE ON FREQUENCIES OF VIBRATION OF SANDWICH PLATE

1x1 and 2x2 meshing are considered for present study. Considering other parameters constant, increase in core thickness increases natural frequency of the sandwich plate. Table 8.5 shows the natural frequencies at different core thickness. Figure 8.4 shows the effect of increase of the core thickness on the natural frequency considering 1x1 mesh size. Difference of the natural frequency is more at higher mode with respect to first mode.

Table 8.5 Frequencies for Different Core Thickness of Sandwich plate

Mesh Size	Thickness of the core (mm)	Frequency (Hz)				
		Mode1	Mode2	Mode3	Mode4	Mode5
1x1	6.35	21.751	43.480	65.241	79.677	86.954
	10	31.265	62.496	93.749	114.51	124.94
	25	61.694	123.23	184.50	225.55	245.73
2x2	6.35	21.670	43.318	64.996	79.378	86.625
	10	31.147	62.251	93.384	114.04	124.44
	25	61.412	122.52	183.41	223.79	243.97

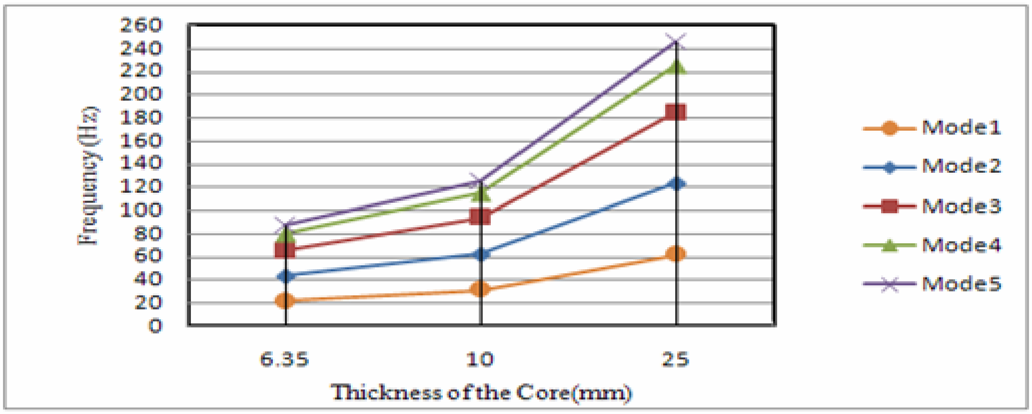


Fig 8.6 Effect of increase of core thickness on frequencies of sandwich panel

8.5.2 EFFECT OF INCREASE IN THICKNESS OF THE FACE SHEETS ON FREQUENCIES OF VIBRATION OF SANDWICH PLATE

1x1 and 2x2 meshing are considered for present study. Considering other parameters constant, increase in face sheets thickness increases natural frequency of the sandwich plate. Table 8.6 shows the natural frequencies at different core thickness. Figure 8.5 shows the effect of increase of the face sheets thickness on the natural frequency considering mesh size 1x1. Difference of the natural frequency is more at higher mode with respect to first mode.

Table 8.6 Frequencies for different face sheets thickness of sandwich panel

Mesh Size	Thickness of the Face sheets (mm)	Frequency (Hz)				
		Mode1	Mode2	Mode3	Mode4	Mode5
1x1	.3	20.494	40.970	61.475	75.087	81.939
	.4064	21.751	43.480	65.241	79.677	86.954
	.5	22.607	45.193	67.810	82.823	90.382
2x2	.3	20.417	40.814	61.239	74.790	81.618
	.4064	21.670	43.318	64.996	79.378	86.625
	.5	22.522	45.021	67.552	82.499	90.031

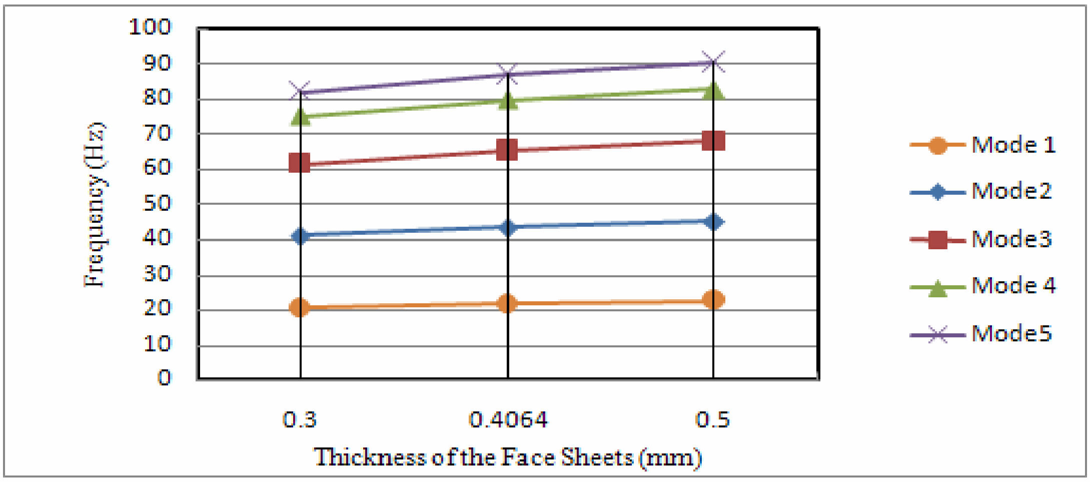


Fig 8.7 Effect of increase of face sheets thickness on frequencies of sandwich panel

8.5.3 THE EFFECT OF INCREASE OF DENSITY OF THE CORE ON THE NATURAL FREQUENCY OF SANDWICH PLATE

Study of variation of natural frequencies due to increase of density of core of simply supported sandwich plate are analyzed by FEA (ANSYS).Table 8.7 shows the different modes of natural frequencies for different densities of the core [16]. Figure 8.6 shows the graph between natural frequencies and densities of the core. Considering other parameters constant, increase in density of the core decreases natural frequency of the sandwich plate.

Increase in density is more effective for higher modes of vibration of simply supported sandwich panel.

Table 8.7 Natural Frequencies of Sandwich Plate at Different Densities of Core

S.No.	Density of Core (kg/m³)	Frequency (Hz)				
		Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
1	121.79	21.670	43.318	64.996	79.378	86.625
2	126	21.575	43.128	64.711	79.029	86.245
3	129	21.508	42.993	64.509	78.784	85.977
4	134.4	21.389	42.755	64.152	78.347	85.500
5	198.4	20.112	40.204	60.324	73.671	80.398

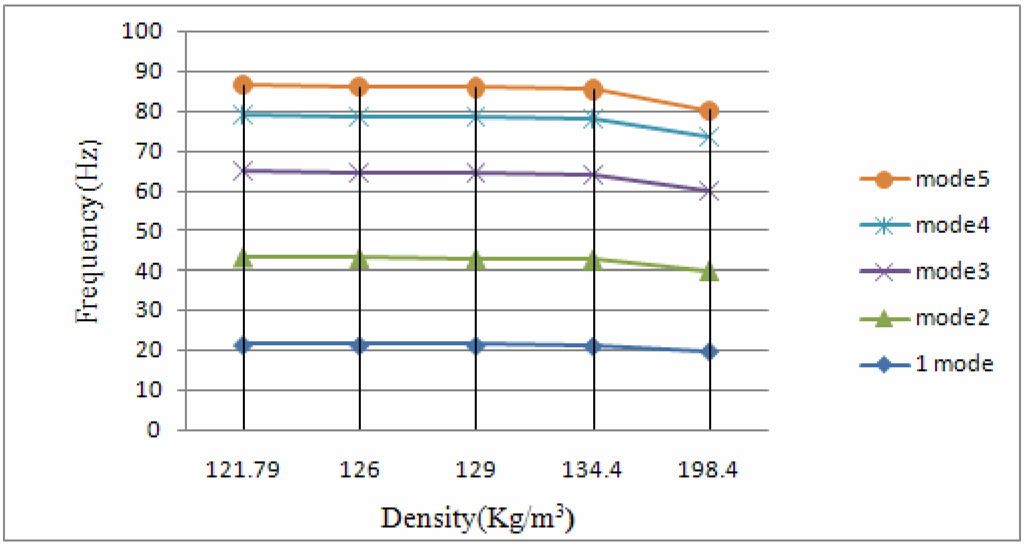


Fig 8.8 Effect of increase of density on natural frequencies of sandwich panel

8.5.4 EFFECT ON THE SANDWICH PLATE'S NATURAL FREQUENCIES OF INCREASING THE CORE DEPTH AS A PERCENTAGE OF ITS TOTAL THICKNESS, WHILST MAINTAINING A CONSTANT MASS

Let us consider

$$h_1 = h_3 \qquad \textbf{... (8.1)}$$

$$h_2 = \alpha(2h_1 + h_2) \qquad \textbf{... (8.2)}$$

Keeping the mass per unit area constant of equivalent solid plate (made of face sheet), total mass of sandwich plate is represented by

$$(2\rho_1 h_1 + \rho_2 h_2) = 19.826 \qquad \textbf{... (8.3)}$$

where

h_1 and h_3 = Thickness of top and bottom face sheets in meter
h_2 = Depth of the core in meter
α = Alpha=Fraction of the core depth relative to the total thickness
ρ_1 = Density of the face sheets in Kg/m^3
ρ_2 = Density of the face sheets in Kg/m^3

From above equations,

$$h_1 = \frac{9.913(1-\alpha)}{[\rho_1(1-\alpha) + \rho_2\alpha]} \qquad \textbf{... (8.4)}$$

By varying Alpha between 0.40 and 0.80, it is possible to see what effect the introduction of a particular depth of honeycomb core will have on the natural frequencies of sandwich plate. Table 8.8 gives natural frequencies for different values of alpha. Figure 8.7 shows the first five frequencies plotted against alpha**.** The rise in frequency in all five modes is due to increase in the percentage depth of the core. This is due to the relative increase in the flexural stiffness of the plate caused by the increasing depth of sandwich core.

To raise the fundamental frequency of the equivalent solid plate (15.426 Hz) without increasing the structural mass, it would be related to replace the equivalent solid plate by a sandwich plate - the fundamental frequency could be doubled by making the thickness of both face plates 3.4793 mm and the core depth 4.639 mm (corresponding to alpha = 0.4), or increased 6-fold by making the thickness of both face plates 3.04 mm and the core depth 24.36 mm (corresponding to alpha = 0.80).

Table 8.8 Frequencies for different values of alpha

Alpha	Frequency(Hz)				
0.4	30.627	61.219	91.851	112.17	122.41
0.5	38.099	76.142	114.22	139.48	152.2
0.6	48.841	97.582	146.34	178.67	194.95
0.7	65.368	130.52	195.63	238.74	260.44
0.8	93.844	187.1	279.97	341.34	372.16

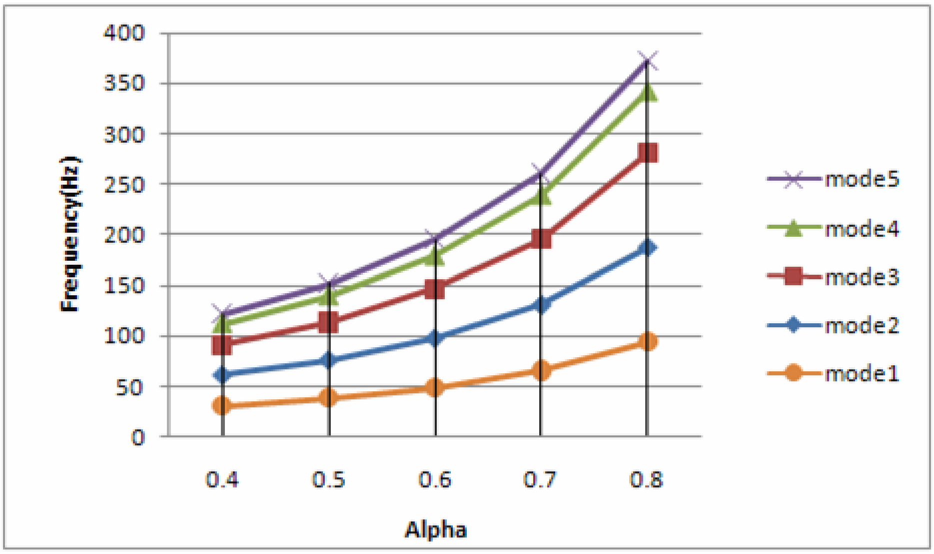


Fig 8.9 Effect of increase of alpha on natural frequency of sandwich panel

CONCLUSION AND FUTURE WORK

The comparison shows that the experimental values of simply supported honeycomb core sandwich panel differ somewhat with FEA (ANSYS) and analytical values. Error in sandwich panel is within 5% range. This can be attributed to the fact that many properties of sandwich panel which are not given and hence assumed during FEA (ANSYS) and analytical calculations.

Convergence analysis has been done by finite element model. The uniformly converging results of free vibration of simply supported sandwich panel assure the accuracy and correctness of the present analysis.

Modal analysis of simply supported aluminium core sandwich plate and same dimensions equivalent face plate shows that sandwich plate having 1.4 times higher fundamental frequency than equivalent face plate. Difference in frequency is more at higher modes. Increase in frequency is due to increase in flexural stiffness of the plate.

Parameter studies have been carried out using the finite element model to investigate the effects of changing thickness of the core and face sheets and variation of density of the core of the sandwich structure on the natural frequencies of free vibration.

Increase in thickness of core increases natural frequency and increase is more at higher modes. Increase in density of the core decreases the natural frequency of the sandwich plate. Theoretically natural frequency is inversely proportional to density of the sandwich plate hence density increase natural frequency decreases. A detailed parameter study is carried out that examined the effect on the natural frequencies of a simply supported sandwich panel by increasing the core depth as a percentage of its total thickness, while maintaining a constant mass. To raise the fundamental frequency of the equivalent solid plate (15.426 Hz) without increasing the structural mass, it would be related to replace the equivalent solid plate by a sandwich plate - the fundamental frequency could be doubled by making the thickness of both face plates 3.4793 mm and the core depth 4.639 mm (corresponding to alpha = 0.4).

Present analysis is used in ALH and LCA for reducing the weight of the parts (tail boom, main doors, fuselage panel, rotor blades, wings etc) and increase the frequency of the parts above the frequency of the ALH and LCA (15 Hz).

REFERENCES

1. Lok T. S. and Cheng Q. H., “Free vibration of clamped orthotropic sandwich panel.” Journal of Sound and &vibration. Volume 229, No. 2, (2000): p. 311-327.

2. Yuan W. X. and Dawe D. J., “Free vibration of sandwich plates with laminated faces.” Int. J. Numer. Meth. Engng. Volume 54, (2002): p. 195–217.

3. F J Plantema. Sandwich Construction. New York: Wiley, 1966.

4. H G Allen. Analysis and Design of Structural Sandwich Panels. Pergamon: Oxford, 1969.

5. Scipio L. Albert. Structure Design concepts. Washingoton D. C.: NASA SP-5039, 1967.

6. Pflug Jochen and Verpoest Ignaas, “Sandwich Materials Selection Charts.” Journal of Sandwich Structures And Materials. Volume 8, (September 2006): p. 407-421.

7. Zhen W. and Wanji C., “Free vibration of laminated composite and sandwich plates using global–local higher-order theory.” Journal of Sound and Vibration. Volume 298, (2006): p. 333–349.

8. Zhou G., Hill M. and Loughlan J., “Damage Characteristics of Composite Honeycomb Sandwich Panels in Bending under Quasi-static Loading.” Journal of Sandwich Structures and Materials. Volume 8, No. 1, (2006): p. 55-90.

9. Krishnamoorthy C. S. Finite Element Analysis Theory and Programming. New Delhi: Tata Mc Hill Co., 1988.

10. Rao J. S. Dynamics of Plates. New Delhi :Narosa Publishing House, 1999.

11. Jones Robert M. Mechanics of Composite Materials. Washington D.C.: Scripta Book Company, 1975.

12. Bhavikatti S. S. Finite Element Analysis. New Delhi: New Age International (P) Limited , 2005

13. Sinha P. K. Composite Materials and Structures. I.I.T. Kharagpur: Department of Aerospace Engineering, 2006

14. Varadan T. K. Analysis of Plates (Theory and Problems). New Delhi: Narosa Publishing House, 1999.

15. Ashton J. E. Theory of Laminated Plates. Stamford: Technomic Publication, 1970.

16. HexWeb Honeycomb Attributes and Properties, A comprehensive guide to standard

Hexcel honeycomb materials, configurations, and mechanical properties Hexcel Corporation, Pleasanton, California, 1999.

17. Hexcel's Composite Materials for the Aerospace Industry, European Airshow Issue, Hexcel Composites, Stamford, 1998.

18. Ding Yunliang, "Optimum Design of Sandwich costructions." Composite Structures. Volume 25, No. 1, (1987): p. 51-68.